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V. I. Kuvshinov, E. G. Bagashov, V. A. Shaparau, A. V. Kuzmin

Joint Institute for Power and Nuclear Research – Sosny of the National Academy of Sciences of Belarus, Minsk, Belarus

NEW EXPERIMENTAL AND THEORETICAL TASKS IN MODERN PARTICLE AND NUCLEAR PHYSICS

Abstract. We discuss the existence of new phenomena and properties of nonperturbative evolution of color quarks, gluons and other states in stochastic vacuum of quantum chromodynamics, color dissipation and confinement; instability of movement of color particles in the confinement region; appearance of squeezed and entangled states of strongly interacting particles; correlation properties of strong instanton decays; chaos-assisted instanton tunneling; quark-gluon plasma properties description in terms of the Hagedorn bootstrap statistical model.

Keywords: quantum chromodynamics, color confinement, instability of movement, squeezed and entangled states of quarks and gluons, strong instantons, chaos-assisted instanton tunneling, quark – gluon plasma, Hagedorn model

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В. И. Кувшинов, Е. Г. Багашов, В. А. Шапоров, А. В. Кузьмин

Объединенный институт энергетических и ядерных исследований – Сосны Национальной академии наук Беларуси, Минск, Беларусь

НОВЫЕ ЭКСПЕРИМЕНТАЛЬНЫЕ И ТЕОРЕТИЧЕСКИЕ ЗАДАЧИ В СОВРЕМЕННОЙ ФИЗИКЕ ЧАСТИЦ И ЯДЕРНОЙ ФИЗИКЕ

Аннотация. Обсуждается существование новых явлений и свойств непертурбативной эволюции цветных кварков, глюонов и других состояний в стохастическом вакууме квантовой хромодинамики, диссипация цвета и конфайнмент; неустойчивость движения цветных частиц в области конфайнмента; возникновение сжатых и перепутанных состояний сильно взаимодействующих частиц; корреляционные свойства распадов сильных инстантонов; ассистированное хаосом туннелирование инстантонов; описание свойств кварк-глюонной плазмы на языке статистической модели бутстрапа Хэджнорна.

Ключевые слова: квантовая хромодинамика, конфайнмент цвета, неустойчивость движения, сжатые и перепутанные состояния кварков и глюонов, сильные инстантоны, хаос-ассистированное туннелирование инстантонов, кварк-глюонная плазма, модель Хэджнорна.

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I. Colour Dissipation by Propagation Through QCD Vacuum

I.1. Decoherence in quantum systems. The density matrix of some quantum system in the environment is obtained by averaging with respect to degrees of freedom of the environment. Interactions with the environment result in decoherence and loss of quantum superpositions and color. Information on the initial state of the quantum system is lost after sufficiently large time. Quantum decoherence is the loss of coherence or ordering of the phase angles between the components of a system in a quantum superposition. Decoherence occurs when a system interacts with its environment in a thermodynamically irreversible way. This process can be viewed as the loss of information on a system. Dissipation is a decohering process by which quantum states are changed due to entanglement with an external environment.

I.2. Stochastic QCD vacuum. The model of stochastic vacuum of quantum chromodynamics (QCD) is one of the popular phenomenological models which explains quark confinement (Wilson loop decreasing), constant string tension and field configurations around static colour charges [I.1, I.3, I.4]. In this model only the second field correlators are important and the others are negligible (Gauss domination) [I.1].

It has been confirmed by lattice calculation [I.2]. The most important evidence for this is Casimir scaling. The stochastic vacuum approach is based on the assumption that one can calculate vacuum expectation values of gauge invariant quantities as expectation values with respect to some well-behaved stochastic gauge field.

I.3. QCD vacuum as environment. We consider QCD stochastic vacuum as the environment for colour quantum particles. To get the density matrices describing the states of these particles we average over the external QCD stochastic vacuum implementations. Thus, instead of considering nonperturbative dynamics of Yang-Mills fields one introduces external environment and average over its implementations. As a consequence, we obtain decoherence, loss of quantum superpositions and information, and confinement of colour. White objects can be obtained as colourless mixtures of states described by the diagonal density matrix as a result of evolution in the QCD stochastic vacuum treated as an environment.

I.4. Colour decoherence. Consider propagation of heavy spinless colour particle along some fixed path γ. The amplitude of such process is obtained by parallel transport [I.5–I.7]:

$$\partial_\mu \left|\varphi\right\rangle = i\hat{A}_\mu \left|\varphi\right\rangle, \tag{I.1}$$

$$\left|\varphi(\gamma)\right\rangle = P\exp\left(i\int_\gamma dx^\mu \hat{A}_\mu\right)\left|\varphi_{in}\right\rangle, \tag{I.2}$$

where P is the path ordering operator and $\hat{A}_\mu$ is the gauge field vector.

In order to consider mixed states we introduce the colour density matrix, taking into account both colour particle and QCD stochastic vacuum (environment):

$$\hat{\rho}(\gamma\overline{\gamma}) = \left|\varphi(\gamma)\right\rangle\left\langle\varphi(\gamma)\right|. \tag{I.3}$$

Here we average over all implementations of stochastic gauge field (environment degrees of freedom) – and decoherence arises due to interaction with environment. In the model of QCD stochastic vacuum only expectation values of path ordered exponents over closed paths are defined (in order to keep the gauge invariance). Closed path corresponds to a process in which the particle-antiparticle pair is created, propagates and finally annihilates. With the help of (I.1), (I.2), (I.3) we obtain the expression for density matrix [I.5, I.7]:

$$\hat{\rho}(\gamma\overline{\gamma}) = N_c^{-1} + \left(\left|\varphi_{in}\right\rangle\left\langle\varphi_{in}\right| - N_c^{-1}\right)W_{adj}(\gamma\overline{\gamma}). \tag{I.4}$$

Here N_c^{-1} is the inverse number of colours, and $W_{adj}(\gamma\overline{\gamma})$ is the Wilson loop in the adjoint representation. In fundamental representation it is defined as

$$W_{adj}(\gamma\overline{\gamma}) = \mathrm{Tr}\, P\exp\left(\int_{\gamma\overline{\gamma}} i\hat{A}_\mu dx^\mu\right). \tag{I.5}$$

Colour density matrix in colour neutral stochastic vacuum can be decomposed into the pieces that transform under trivial and adjoint representations [I.5, I.7]

$$\hat{\rho}_1 = N_c^{-1}\hat{I} + \rho_1^a \hat{T}_a, \tag{I.6}$$

where $\hat{I}$ is the unit operator. In confinement region Wilson loop decays exponentially with the area spanned by loop, so for the rectangular loop spanning the time interval T and distance R we get:

$$\hat{\rho}(\gamma\overline{\gamma}) = \hat{I}N_c^{-1} + \left(\hat{\rho}_{in} - \hat{I}N_c^{-1}\right)\exp\left(-\sigma_{adj}RT\right), \tag{I.7}$$

$$\hat{\rho}(\gamma : RT \to \infty) = \hat{I}N_c^{-1}, \tag{I.8}$$

where $\sigma_{adj} = \sigma_{fund} G_{adj} G_{fund}^{-1}$ is the string tension in the adjoint representation, G_{adj}, G_{fund} are the eigenvalues of quadratic Casimir operators. Under Gaussian dominance string tension is

$$\sigma_{fund} = \frac{g^2}{2} l_{corr}^2 F^2, \tag{I.9}$$

g is coupling constant, l_{corr} – correlation length in the QCD stochastic vacuum, F – average of the second cumulant of curvature tensor [I.2, I.4]. As follows from (I.8), all colour states are mixed with equal probabilities and all information on initial color state is lost.

I.5. Decoherencerate, purity, von Neumann entropy, information. The decoherence rate of transition from pure colour states to white mixture state can be estimated on the base of purity [I.8]:

$$p = \mathrm{Tr}\,\hat{\rho}^2. \tag{I.10}$$

This characteristic represents the closeness of a quantum state to a pure one. In our case,

$$p = N_c^{-1} + \left(1 - N_c^{-1}\right)\exp\left(-2\sigma_{fund} G_{adj} G_{fund}^{-1} RT\right). \tag{I.11}$$

When RT tends to 0, $p \to 1$, which corresponds to a pure state. When the composition RT tends to infinity, the purity tends to N_c^{-1}, which corresponds to the white mixture [I.7]. The rate of purity decrease is

$$T_{dec}^{-1} = -2\sigma_{fund} G_{adj} G_{fund}^{-1} R. \tag{I.12}$$

Left side of the equation is the inverse characteristic time of decoherence proportional to QCD string tension and distance R. It can be inferred from (I.11) that the larger is the distance between particle and antiparticle, the quicker the initial state tends to white mixture as a result of interaction with the QCD stochastic vacuum. Thus white states can be obtained as a result of decoherence process. The information on quark colour states is lost in hadrons due to interactions between quarks and confining non-Abelian gauge fields. Von Neumann entropy can be used as a measure of the loss of information:

$$S = -\mathrm{Tr}\left(\hat{\rho} \ln \hat{\rho}\right). \tag{I.13}$$

Initial density matrix gives us $S = 0$ and we will have $S = \ln N_c$ for large RT. In order to obtain a more suitable characteristic to work with, we might want to define the information on the basis of the given entropy parameter. As it can be seen, the entropy ranges from 0 (for a pure, unperturbed quantum state, which in our approach corresponds to the maximum amount of quantum information) to $\ln N_c$. So it would be quite natural to define the information measure as

$$I = \frac{\ln N_c - S}{\ln N_c} = 1 - \frac{S}{\ln N_c}. \tag{I.14}$$

So the overall range of this measure is [0, 1]: in case of zero entropy it is equal to 1 and in case of maximum entropy it is equal to 0. The latter case corresponds to the asymptotically big values of RT (Wilson loop area). Thus, as we might see, during the interaction with QCD vacuum the entropy increases and the information is being lost due to interactions between quarks and confining non-Abelian gauge fields.

I.6. Instability of colour particle motion in confinement region. Wilson loop definition in QCD is similar with the definition of fidelity, the quantity which describes the stability of quantum motion of the particles [I.9]. Using the analogy between the theory of gauge fields and the theory of holonomic quantum computation [I.9–I.11], we can define the fidelity as an integral over the closed loop, with particle traveling from point x to the point y:

$$f = \langle \varphi_{in} | P \exp\left(\int i\hat{A}_\mu dx^\mu \right) | \varphi_{in} \rangle. \tag{I.15}$$

The final expression for the fidelity of the particle moving stochastic vacuum is

$$f = \exp\left(-\frac{1}{2} g^2 l_{corr}^2 F^2 S_\gamma \right). \tag{I.16}$$

Thus fidelity for colour particle moving along the contour decays exponentially with the surface spanned over the contour, the decay rate being equal to the string tension (I.9). The motion becomes more and more instable with the increase in the area.

I.7. Order to chaos transition, critical energy, Higgs mass. The increasing of instability of motion in the confinement region is also connected with existence of chaotic solutions of Yang-Mills field [I.5, I.12], possible chaos onset [I.13]. Yang-Mills fields already at classical level show inherent chaotic dynamics and have chaotic solutions [I.12, I.13]. It has been shown that Higgs bosons and its vacuum quantum fluctuations regularize the system and lead to the emergence of order-chaos transition at some critical energy [I.14, I.15]:

$$E_c = \frac{3\mu^4}{64\pi^2} \exp\left(1 - \frac{\alpha}{g^4} \right). \tag{I.17}$$

Here μ is mass of Higgs boson, α is its self interaction coupling constant, *g* is coupling constant of gauge and Higgs fields. Important here is the value of mass of Higgs boson. In the region of confinement there exists the point of order-chaos transition where the fidelity begin to decrease exponentially. This connects the properties of stochastic QCD vacuum, Higgs boson mass and self interaction coupling constants.

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II. Gluon Squeezed and Entangled States in QCD

Many experiments are devoted to hadronic jet physics, since in particular, detailed studies of jets are important for better understanding and testing both perturbative and nonperturbative QCD. Predictions of the PQCD are limited by small effective coupling $\alpha(Q^2) < 1$ and nonperturbative phase is usually taken into account either through a constant factor which relates partonic features with hadronic ones (within local parton-hadron duality-LPHD) or through the application of various phenomenological models of hadronization. Usually both stages should be taken into account. So as the width of the multiplicity distribution (MD) according to the predictions only of PQCD is larger than the experimental one. The discrepancies between theoretical calculations and experimental data suggest that after perturbative stage the quark-gluon cascade undergoes non-perturbative evolution after that hadronization effects come into play. For example, such a contribution to the multiplicity distribution can be made in the form of the sub-Poissonian distribution [II.3, II.4]. Calculations performed within PQCD [II.5, II.6] show that multiplicity distribution at the end of the perturbative cascade is close to a negative binomial distribution. At the same time, gluon MD in the range of the small transverse momenta (thin ring of jet) is Poissonian [II.7]. Thus parton MD in the whole jet at the end of the perturbative cascade can be represented as a combination of Poissonian distributions each of which corresponds to a coherent state. Studying a further evolution of gluon states at the non-perturbative stage of jet evolution we obtain new gluon states that are squeezed states (SS) [II.8–II.11]. These states are formed as a result of nonperturbative self-interaction of the gluons expressed by nonlinearities of Hamiltonian. In this paper we prove that nonperturbative stage of jet evolution can be source of a gluon SS by analogy with nonlinear medium for photon SS [II.12–II.15]. Squeezed states posses uncommon properties: they display a specific behaviour of the factorial and cumulant moments [II.16] and can have both sub-Poissonian and super-Poissonian statistics corresponding to antibunching and bunching of photons. Moreover oscillatory behaviour MD of photon SS is differentiated from Poissonian and Negative binomial distributions (NBD). Because of analogy between photon and gluon, MD of gluon SS must have oscillations and using Local parton hadron duality (LPHD) we can compare derived gluon MD with hadron MD. It is clear that in this case behaviour of hadron MD in jet events is differentiated from NBD and this fact is confirmed by experiments for $pp, p\bar{p}$-collisions [II.17–II.19]. As pion contribution to the jet events is dominant, calculation of the pion characteristics is an important task at investigation of the different physical phenomena. Within LPHD we can calculate correlation characteristics extend to the pions. It is reasonable that the distinctive features of the gluon squeezed state correlations will be reflected in the pion correlation behavior. In this work using LPHD we estimate nonperturbative contribution of the gluon squeezed states without taking into account its polarization to the pion correlation functions in the jet narrow ring. At the same time two-mode photon SS [II.13, II.20] in the limit of infinite squeezing are isomorphic to the Bell states [II.21] which have been introduced in relation to the Einstein-Podolsky-Rosen (EPR) paradox [II.22] and they are one of the examples of the entangled states for two polarizations. The states

$$
\begin{aligned}
\left|\Phi^{+}\right\rangle &= \left(|\updownarrow\rangle_1|\updownarrow\rangle_2 + |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2\right)\Big/\sqrt{2}, \qquad \left|\Phi^{-}\right\rangle = \left(|\updownarrow\rangle_1|\updownarrow\rangle_2 - |\leftrightarrow\rangle_1|\leftrightarrow\rangle_2\right)\Big/\sqrt{2}, \\
\left|\Psi^{+}\right\rangle &= \left(|\updownarrow\rangle_1|\leftrightarrow\rangle_2 + |\leftrightarrow\rangle_1|\updownarrow\rangle_2\right)\Big/\sqrt{2}, \qquad \left|\Psi^{-}\right\rangle = \left(|\updownarrow\rangle_1|\leftrightarrow\rangle_2 - |\leftrightarrow\rangle_1|\updownarrow\rangle_2\right)\Big/\sqrt{2}
\end{aligned}
\tag{II.1}
$$

are the basis of the Bell states. Each of these entangled states, for example, $\left|\Phi^{\pm}\right\rangle$, has a uncommon property: if one photon is registered with defined polarization (for example, with polarization $\updownarrow$), the other photon immediately becomes opposite polarized (longitudinal polarization). If we suppose that two-mode gluon entangled states with two different colours can lead to $q\bar{q}$-entangled states then interaction of the quark entangled states with stochastic vacuum has a remarkable property, namely, as soon as some measurement projects one quark onto a state with definite colour, the another quark also immediately obtains opposite colour that leads to coupling of quark-antiquark pair, string tension inside $q\bar{q}$-pare and free propagation of colourless hadrons.

II.1. Gluon single-mode squeezed states in QCD jet model. The solution of the Schrödinger evolution equation for small time t

$$\left|\mathrm{f}\right\rangle \simeq \left|\mathrm{in}\right\rangle - itV\left|\mathrm{in}\right\rangle \tag{II.2}$$

provides a possibility to observe an evolution of an initial state vector $\left|\mathrm{in}\right\rangle$ for small time. Here the Hamiltonian of gluon self-interaction for the jet ring model (Fig. 1) of thickness $d\theta$ in the momentum representation can be represented in the form [II.9–II.11]

$$V = \frac{k_0^4}{4(2\pi)^3}\left(1-\frac{q_0^2}{k_0^2}\right)^{3/2} g^2\pi f_{abc} f_{ade}\left\{\left(2-\frac{q_0^2}{k_0^2}\right)\left[a_{1212}^{bcde}+a_{1313}^{bcde}\right]+a_{2323}^{bcde}+\right.$$

$$\left.+\frac{\sin^2\theta}{2}\left(1-\frac{q_0^2}{k_0^2}\right)\left[2a_{2323}^{bcde}-a_{1212}^{bcde}-a_{1313}^{bcde}\right]\right\}\sin\theta d\theta. \tag{II.3}$$

Here a_{lmlm}^{bcde}=$a_l^{b+}a_m^{c+}a_l^d a_m^e + a_l^{b+}a_m^c a_l^{d+}a_m^e + a_l^b a_m^{c+}a_l^{d+}a_m^e + h.c.$, $a_l^b\left(a_l^{b+}\right)$ is the operator annihilating (creating) a gluon of colour b and vector component l, q_0^2 and k_0 are correspondingly the virtuality and energy of the gluon at the end of the perturbative cascade, g is the coupling constant, f_{abc} are the structure constants of the $SU_c(3)$ group, θ is the angle between a gluon momentum **k** and its progenitor ($0 \le \theta \le \theta_{\max}, \theta_{\max}$ is half of the opening angle of the jet cone).

Since product of the gluon coherent states with different colour and vector indices corresponds to Poissonian distribution of the multimode gluon states in thin ring of jet [II.7], present state vector may be considered as initial state vector $\left|\mathrm{in}\right\rangle$ prepared by the perturbative stage, that is

$$\left|\mathrm{in}\right\rangle \equiv \left|\alpha\right\rangle = \prod_{c=1}^{8}\prod_{l=1}^{3}\left|\alpha_l^c\right\rangle. \tag{II.4}$$

Gluon coherent state vector $\left|\alpha_l^c\right\rangle$ is the eigenvector of the corresponding annihilation operator a_l^c with the eigenvalue α_l^c which can be written in terms of the gluon coherent field amplitude $|\alpha_l^c|$ and phase γ_l^c of the given gluon field $\alpha_l^c = |\alpha_l^c| e^{i\gamma_l^c}$. Thus evolution of this initial state vector within a small interval of time t is defined according (II.2) as

$$\left|\mathrm{f}\right\rangle = \prod_{c=1}^{8}\prod_{l=1}^{3}\left|\alpha_l^c(t)\right\rangle \simeq (1-itV)\prod_{c=1}^{8}\prod_{l=1}^{3}\left|\alpha_l^c\right\rangle. \tag{II.5}$$

In this case the time is reckoned up from the beginning of nonperturbative stage and the Hamiltonian of the gluon self-interaction V in the jet ring is determined by formula (I.3). The explicit form of the evolved state vector $\left|\mathrm{f}\right\rangle$ is given in [II.10, II.23]. The Hamiltonian of the gluon self-interaction V in the jet ring includes the squares of the creation and annihilation operators for gluons with specified

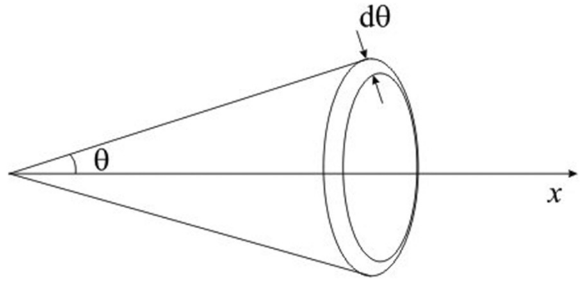


Fig. II.1. Jet ring

colour and vector indices. As is known from quantum mechanics and quantum optics, the presence of such structure in the Hamiltonian and, consequently, in the evolution operator is a necessary condition for emergence of squeezed states [II.14], since the unitary squeezing operator involves quadratic combinations of the creation and annihilation operators

$$S(z) = \exp\left\{\frac{z^*}{2}a^2 - \frac{z}{2}(a^+)^2\right\}, \quad \text{(II.6)}$$

where $z = re^{i\vartheta}$ is an arbitrary complex number, r is a squeeze factor, phase ϑ defines the direction of squeezing maximum [II.14]. In order to verify whether the final gluon state vector describes the single-mode SS, it is necessary to introduce the phase-sensitive Hermitian operators $(X_l^b)_1 = \left[a_l^b + (a_l^b)^+\right]/2$ and $(X_l^b)_2 = \left[a_l^b - (a_l^b)^+\right]/2i$ by analogy with quantum optics and to establish conditions under which the variance of one of them can be less than the variance of a coherent state. Condition of the single-mode squeezing for gluons is expressed in the form of the inequalities [II.9–II.11]

$$\left\langle \left(\Delta(X_l^b)_{\substack{1\\2}}\right)^2 \right\rangle = \left\langle N\left(\Delta(X_l^b)_{\substack{1\\2}}\right)^2 \right\rangle + \frac{1}{4} < \frac{1}{4} \quad \text{or} \quad \left\langle N\left(\Delta(X_l^b)_{\substack{1\\2}}\right)^2 \right\rangle < 0. \quad \text{(II.7)}$$

Here N is the normal-ordering operator

$$\left\langle N\left(\Delta(X_l^b)_{\substack{1\\2}}\right)^2 \right\rangle = \frac{1}{4} \pm \left\{\left[\left\langle (a_l^b)^2\right\rangle - \left\langle a_l^b\right\rangle^2\right] \pm \left[\left\langle (a_l^{b+})^2\right\rangle - \left\langle a_l^{b+}\right\rangle^2\right] + 2\left[\left\langle a_l^{b+}a_l^b\right\rangle - \left\langle a_l^{b+}\right\rangle\left\langle a_l^b\right\rangle\right]\right\}. \quad \text{(II.8)}$$

The expectation values of the creation and annihilation operators for gluons with specified colour and vector components are taken for the vector $|\mathrm{f}\rangle$ (II.5). Let us consider the specific case where the colour index is $b = 1$ and the vector index l is arbitrary. Then we have

$$\left\langle N\left(\Delta(X_l^1)_{\substack{1\\2}}\right)^2 \right\rangle = \pm 4\pi u_2 t \sin\theta d\theta \{(1+u_1)\left[\delta_{l1}(Z_{33}+Z_{22}) + (1-\delta_{l1})Z_{11}\right] + \left[\delta_{l2}Z_{33} + \delta_{l3}Z_{22}\right] +$$
$$+ u_1 \sin^2\theta\left[-\frac{1}{2}\delta_{l1}(Z_{22}+Z_{33}) + \delta_{l2}(Z_{33} - \frac{1}{2}Z_{11}) + \delta_{l3}(Z_{22} - \frac{1}{2}Z_{11})\right]\right\}. \quad \text{(II.9)}$$

Here $Z_{mn} = \sum_{k=2}^{7}\left\langle (X_m^k)_1\right\rangle\left\langle (X_n^k)_2\right\rangle \quad (m, n = 1,2,3)$.

In final state being consideration fluctuations of one of the squared components of the gluon field, $\Delta(X_l^1)_2$, are less than those in the initial coherent state under the following conditions: $\left\langle (X_m^k)_1\right\rangle < 0, \left\langle (X_m^k)_2\right\rangle < 0$ or $\left\langle (X_m^k)_1\right\rangle > 0, \left\langle (X_m^k)_2\right\rangle > 0 \quad (k \neq 1, m \neq l)$. In this case we have phase-squeezed gluon states by analogy with quantum optics [II.13, II.14].

If the conditions $\left\langle (X_m^k)_1\right\rangle > 0, \left\langle (X_m^k)_2\right\rangle < 0$ or $\left\langle (X_m^k)_1\right\rangle < 0$ and $\left\langle (X_m^k)_2\right\rangle > 0 \quad (k \neq 1, m \neq l)$ are satisfied, fluctuations in another squared component of the gluon field, $\Delta(X_l^1)_1$, will be less in the final state vector $\prod_{c=1}^{8}\prod_{l=1}^{3}\left|\alpha_l^c(t)\right\rangle$ than in the coherent state. In this case we come to the amplitude-squeezed gluon states (as in the case of photons [II.13, II.14]). Since at small value of the squeeze factor we have

$$\left\langle N\left(\Delta(X_l^b)_{\substack{1\\2}}\right)^2 \right\rangle = \mp\frac{1}{2}r_l^b \cos\vartheta. \quad \text{(II.10)}$$

Evidently that $\left\langle N\left(\Delta(X_l^b)_2\right)^2\right\rangle < 0$ (phase-squeezed gluon states) if $\frac{\pi}{2} < \vartheta < \frac{3\pi}{2}$ and $\left\langle N\left(\Delta(X_l^b)_1\right)^2\right\rangle < 0$ (amplitude-squeezed gluon states) if $-\frac{\pi}{2} < \vartheta < \frac{\pi}{2}$. Taking into account formula (II.9) and (II.10) the expression for the squeezing parameter in terms of the amplitude and phase of the gluon coherent states $\left(\alpha_l^b = \left|\alpha_l^b\right| e^{i\gamma_l^b}\right)$ is written as

$$r_l^1 \cos\vartheta = -8\pi u_2 t \sin\theta d\theta \sum_{k=2}^{7} \left\{ (1+u_1 - \frac{u_1}{2}\sin^2\theta) \left[\delta_{l1} \sum_{n=2}^{3} |\alpha_n^k|^2 \sin(2\gamma_n^k) + (1-\delta_{l1}) |\alpha_1^k|^2 \sin(2\gamma_1^k) \right] + \right.$$
$$\left. +(1+u_1\sin^2\theta)\left[\delta_{l2} |\alpha_3^k|^2 \sin(2\gamma_3^k) + \delta_{l3} |\alpha_2^k|^2 \sin(2\gamma_2^k) \right] \right\}. \quad \text{(II.11)}$$

Obviously that effect of the single-mode squeezing is absent $\left(r_l^1 \cos\vartheta = 0\right)$ then the initial gluon coherent fields are either real $\left(\gamma_n^k = 0, n \neq l, k \neq 1\right)$ or imaginary $\left(\gamma_n^k = \pi/2, n \neq l, k \neq 1\right)$. Similar conclusions will also be valid for a gluon field with other colour indices. Thus, the evolved vector

$$\prod_{c=1}^{8}\prod_{l=1}^{3}\left|\alpha_l^c(t)\right\rangle \quad \text{(II.12)}$$

describes the single-mode squeezed state of gluons that are produced at the nonperturbative stage of the jet evolution within a small interval of time t.

II.2. Gluon squeezed states contribution of the to the pion correlation functions in QCD jet. Normalized second order correlation function for the gluons with colour b and vector component l is

$$K_{l(2)}^b(\theta_1,\theta_2) = \frac{\rho_{l(2)}^b(\theta_1,\theta_2)}{\rho_{l(1)}^b(\theta_1)\rho_{l(1)}^b(\theta_2)} - 1, \quad \text{(II.13)}$$

where

$$\left.\begin{aligned} \rho_{l(1)}^b(\theta) &= \left\langle f(\theta,t) \,|\, a_l^{b+} a_l^b \,|\, f(\theta,t) \right\rangle, \\ \rho_{l(2)}^b(\theta_1,\theta_2) &= \left\langle f(\theta_2,t), f(\theta_1,t) \,|\, a_l^{b+} a_l^{b+} a_l^b a_l^b \,|\, f(\theta_1,t), f(\theta_2,t) \right\rangle \end{aligned}\right\}, \quad \text{(II.14)}$$

$\left|f(\theta,t)\right\rangle$ is a final state vector defined within small time interval as

$$\left|\mathrm{f}\right\rangle \equiv \prod_{c=1}^{8}\prod_{l=1}^{3}\left|\alpha_l^c(\theta_1,t), \alpha_l^c(\theta_2,t)\right\rangle \simeq (1 - itV)\prod_{c=1}^{8}\prod_{l=1}^{3}\left|\alpha_l^c(\theta_1), \alpha_l^c(\theta_2)\right\rangle. \quad \text{(II.15)}$$

By taking the expectation values over the final vector[1] and taking into account correlations in jet thin ring thickness of which is defined by Δ, the slope angle of which is defined by θ, we obtain the explicit form of the normalized second-order correlation function for squeezed gluon states

$$K_{l(2)}^b(\theta,) = -M_1(\theta,\Delta) \Big/ \left\{ \left|\alpha_l^b\right|^4 - 2\left|\alpha_l^b\right|^2 M_1(\theta,\Delta) + M_2(\theta,\Delta) \right\}. \quad \text{(II.16)}$$

As a approximation of the external field ($\alpha_l^b = |\alpha| e^{i\gamma_1}$ and $\alpha_l^c = |\beta| e^{i\gamma_2}$ at $c \neq b$ for $\forall l$, $\gamma_1 - \gamma_2 = \vartheta/2 + \pi/4$) functions $M_1(\theta,\Delta)$ and $M_2(\theta,\Delta)$ have the next forms

[1] That this vector also describes squeezed gluon states can be proven by verifying the squeezing condition.

$$M_1(\theta,\ \Delta) = 24\, t\, u_2\, \pi|\alpha|^2|\beta|^2 \sin\left(\vartheta + \frac{\pi}{2}\right)\{(1+\delta_{l1})(2+u_1-\delta_{l1}) - u_1(3\delta_{l1}-1)(\sin\theta + \Delta\cos\theta)\sin\theta\}, \quad (II.17)$$

$$M_2(\theta,\ \Delta) = 80\, t\, u_2\, \pi|\alpha|^3|\beta|^3 \sin\left(\frac{\vartheta}{2} + \frac{\pi}{4}\right)\{(1+\delta_{l1})(2+u_1-\delta_{l1}) - u_1(3\delta_{l1}-1)(\sin\theta + \Delta\cos\theta)\sin\theta\}. \quad (II.18)$$

Here u_1,u_2 are given in [II.23], ϑ defines the direction of squeezing maximum [II.14] (the squeezing condition is fulfilled at $\vartheta \neq \pi/2, 3\pi/2$). We use LPHD, summarize over color and vector components with weights ω_l^b at transition to pion correlations. In this case nonperturbative contribution to the pion correlation functions appear as the second order normalized correlation function

$$K_{(2)}(\theta,\Delta) = -\sum_{l=1}^{3}\sum_{b=1}^{8}\omega_l^b M_1(\theta,\Delta) \Bigg/ \left\{\sum_{l=1}^{3}\sum_{b=1}^{8}\omega_l^b\left|\alpha_l^b\right|^4 - 2\sum_{l=1}^{3}\sum_{b=1}^{8}\omega_l^b\left|\alpha_l^b\right|^2 M_1(\theta,\Delta) + \sum_{l=1}^{3}\sum_{b=1}^{8}\omega_l^b M_2(\theta,\Delta)\right\}. \quad (II.19)$$

Let us perform a comparative analysis of the correlation function (II.19) for gluon squeezed states and the corresponding function for photon squeezed states, in quantum optics [II.15]

$$K_{(2)} = g^{(2)} - 1 = \frac{\left\langle \hat{a}^+\hat{a}^+\hat{a}\,\hat{a}\right\rangle}{\left\langle \hat{a}^+\hat{a}\right\rangle^2} - 1. \quad (II.20)$$

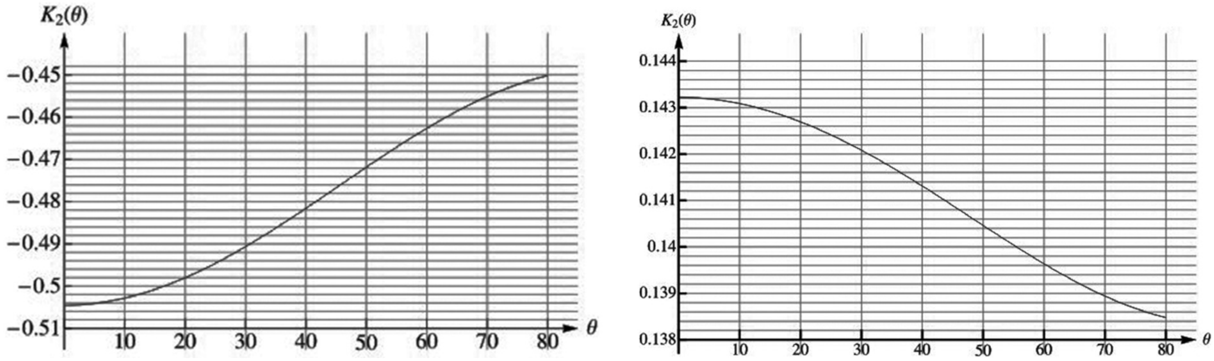
Here the expectation values are taken over the evolved state vector at the time t. If the correlation function is positive, occurs photon bunching (super-Poissonian distribution); otherwise $\left(K_{(2)} < 0\right)$, we have photon antibunching (sub-Poissonian distribution) [II.13, II.15]. For a coherent field obeying Poissonian statistics, the normalized second-order correlation function vanishes $\left(K_{(2)} = 0\right)$.

For the photon squeezed coherent states $\left|[\alpha,\xi]\right\rangle = \hat{S}(\xi)\hat{D}(\alpha)\left|0\right\rangle$ [II.13] the corresponding correlation function has the form (at small values of the squeezing parameter r_l)

$$K_{(2)} = -\frac{r\left[\alpha^2 e^{-i\delta} + (\alpha^*)^2 e^{i\delta}\right]}{|\alpha|^4 - 2r|\alpha|^2\left[\alpha^2 e^{-i\delta} + (\alpha^*)^2 e^{i\delta}\right]}. \quad (II.21)$$

In contrast to the correlation function for squeezed photon states, the corresponding function for the squeezed gluon states, $K_{(2)}(\theta,\Delta)$, includes, as follows from (II.19), the $M_2(\theta,\Delta)$ which appears because the Hamiltonian of the gluon self-interaction involves a nonlinear combination of the creation and annihilation operators of gluons with different colours and vector components. Nonperturbative contribution in the angle dependence of the pion correlation function is graphically investigated (Fig. II.2) at the parameters $q_0^2 = 1\ \text{GeV}^2$; $g^2 = 4\pi$; $k_0 = \sqrt{s}\big/\left(2\left\langle n_{\text{gluon}}\right\rangle\right)$; $\sqrt{s} = 91$ GeV; $\omega_1^b = \omega_2^b$, $\omega_3^b = 0$; $\left\langle n_{\text{gluon}}\right\rangle = |\alpha|^2 + 7|\beta|^2$ is an experimental value which applies restriction on the amplitude values of the coherent fields (investigated and external fields), for the two-jet events – $\left\langle n_{\text{gluon}}\right\rangle = 10$.

Taking into account the nonperturbative cophase squeezed states of the soft gluons ($\vartheta = 0$) we obtain the angular correlations which lie in a negative area. In this case we observe the effect of the pion antibunching with corresponding sub-poissonian distribution (Fig. II.2, *a*) at any values of the coherent field amplitudes. In the case of the nonperturbative antiphase squeezed states of the soft gluons ($\vartheta = \pi$) we have the positive angular correlations. In this case there is the effect of the pion bunching with corresponding super-poissonian distribution (Fig. II.2, *b*) at any values of the coherent field amplitudes. As derived conclusions are similar in case of the photon squeezed states we can say about finding of decreasing (or increasing) of the pion correlations (Fig. II.2) that may be evidence of the gluon squeezed states at the nonperturbative evolution of a hadron jet.



a *b*

Fig.II.2. Angular dependence of the pion correlation function at $\langle n_{\text{gluon}}\rangle = 10$ and (*a*) –(ϑ = 0); (*b*) – ϑ = π

II.3. Gluon two-mode squeezed states in QCD. In order to verify whether the gluon state vector describes the two-mode squeezed state on colours *h* and *g*, it is necessary to introduce the phase-sensitive Hermitian operators $(X_l^{h,g})_1 = \left[a_l^h + a_l^g + a_l^{h+} + a_l^{g+}\right]/(2\sqrt{2})$ and $(X_l^{h,g})_2 = \left[a_l^h + a_l^g - a_l^{h+} - a_l^{g+}\right]/(2i\sqrt{2})$ by analogy with quantum optics [II.13] and to establish conditions under which the variance of one of them can be less than the variance of a coherent state. Here $a_l^h, a_l^g\,(a_l^{h+}, a_l^{g+})$ are the operators annihilating (creating) of gluons with colours $h, g = \overline{1,8}$ and vector indexes $l = \overline{1,3}.$ The condition of the two-mode squeezing of the gluons with different colours *h,g* is expressed in the form of the inequality

$$\left\langle N\left(\Delta(X_l^{h,g})_{\substack{1\\2}}\right)^2\right\rangle < 0. \qquad \text{(II.22)}$$

The expectation values of the creation and annihilation operators for gluons with specified colour and vector index are taken over the vector $|\mathrm{f}\rangle$

$$|\mathrm{f}\rangle \simeq \mathrm{in} - it H_{\mathrm{I}}(t_0)|\mathrm{in}\rangle, \qquad \text{(II.23)}$$

where $H_{\mathrm{I}}(t_0) = H_{\mathrm{I}}^{(3)}(t_0) + H_{\mathrm{I}}^{(4)}(t_0)$ is the Hamiltonian three-gluon $\left(H_{\mathrm{I}}^{(3)}\right)$ and four-gluon $\left(H_{\mathrm{I}}^{(4)}\right)$ self-interactions which explicit forms are given in [II.25] in momentum representation, $|\mathrm{in}\rangle$ is an initial state vector of the virtual gluon field (II.4). Averaging the annihilation and creation operators $a_l^h, a_l^g, a_l^{h+}, a_l^{g+}$ over the evolved vector $|\mathrm{f}\rangle$ vector we write the two-mode squeezing condition in the form

$$\left\langle N\left(\Delta(X_l^{h,g})_{\substack{1\\2}}\right)^2\right\rangle = \pm\frac{it}{8}\Big\{\langle\alpha|[[H_{\mathrm{I}}(0), a_l^{h+}], a_l^{h+}]|\alpha\rangle + \langle\alpha|[[H_{\mathrm{I}}(0), a_l^{g+}], a_l^{g+}]|\alpha\rangle +$$
$$+ 2\langle\alpha|[[H_{\mathrm{I}}(0), a_l^{h+}], a_l^{g+}]|\alpha\rangle - h.c.\Big\} < 0. \qquad \text{(II.24)}$$

It is easy to show that the three-gluon self-interaction (as for single-mode squeezing of the gluons) does not lead to squeezing effect since

$$\left.\begin{array}{lll}\left[\left[H_{\mathrm{I}}^{(3)}(0), a_l^{h+}\right], a_l^{h+}\right] = 0, & \left[\left[H_{\mathrm{I}}^{(3)}(0), a_l^{g+}\right], a_l^{g+}\right] = 0, & \left[\left[H_{\mathrm{I}}^{(3)}(0), a_l^{h+}\right], a_l^{g+}\right] = 0, \\ \left[a_l^g, \left[a_l^h, H_{\mathrm{I}}^{(3)}(0)\right]\right] = 0, & \left[a_l^h, \left[a_l^h, H_{\mathrm{I}}^{(3)}(0)\right]\right] = 0, & \left[a_l^g, \left[a_l^g, H_{\mathrm{I}}^{(3)}(0)\right]\right] = 0.\end{array}\right\} \qquad \text{(II.25)}$$

Thus, only the four-gluon self-interaction can yield a two-mode squeezing effect.

For collinear gluon corresponding squeezing condition is

$$\left\langle N\left(\Delta(X_l^{h,g})_{\frac{1}{2}}\right)^2\right\rangle = \pm t\frac{g^2}{16k_0}(f_{ahb}f_{ahc}+f_{agb}f_{agc}+f_{ahb}f_{agc}+f_{agb}f_{ahc})\sum_{j\neq l}|\alpha_j^b||\alpha_j^c|\sin(\gamma_j^b+\gamma_j^c)<0. \quad (II.26)$$

Here we have taken into account that $\alpha_j^b = |\alpha_j^b| e^{i\gamma_j^b}$ and $\alpha_j^c = |\alpha_j^c| e^{i\gamma_j^c}$. The two-mode squeezing condition is fulfilled for any cases apart from $\gamma_j^b + \gamma_j^c = 0, \pi$. In particular, if all initial gluon coherent fields are real or imaginary then the two-mode squeezing condition is not fulfilled as in the single-mode case.

II.4. Entangled collinear gluon states. At finite squeezed r a continuous variables entangled state is known from quantum optics as a two-mode squeezed state [II.13, II.20]

$$|\mathrm{f}\rangle = S_{12}(r)|0\rangle_1|0\rangle_2 = \frac{1}{\cosh r}\sum_{n=0}^{\infty}(\tanh r)^n|n\rangle_1|n\rangle_2, \quad (II.27)$$

where $S_{12}(r)=\exp\left\{r(a_1^+a_2^+ - a_1a_2)\right\}$ is operator of two-mode squeezing. It is not difficult to demonstrate that the state vector $|\mathrm{f}\rangle$ describes the entangled state. At small value of the squeeze factor we have

$$r = 2\left|\left\langle N\left(\Delta(X_l^{h,g})_{\frac{1}{2}}\right)^2\right\rangle\right|. \quad (II.28)$$

The squeeze factor for the collinear gluons is defined as

$$r = t\frac{g^2}{8k_0}(f_{ahb}f_{ahc}+f_{agb}f_{agc}+f_{ahb}f_{agc}+f_{agb}f_{ahc})\sum_{j\neq l}|\alpha_l^b||\alpha_j^c|\sin(\gamma_j^b+\gamma_j^c). \quad (II.29)$$

From the obtained expression (30) it follows that the squeeze factor is not equal to zero for any cases apart from $\gamma_j^b + \gamma_j^c = 0, \pi$. The dimensionless coefficient

$$y = \left[\frac{|\overline{a_1a_2^+}|^2 + |\overline{a_1\hat{a}_2}|^2}{2(\overline{a_1^+a_1}+1/2)(\overline{a_2^+a_2}+1/2)}\right]^{1/2} \quad (II.30)$$

is the measure of entanglement for two-mode states [II.26], $0 \le y < 1$ (entanglement is not observed when $y = 0$). Averaging the annihilation and creation operators in the expression (31) over the vector $|\mathrm{f}\rangle$ (II.27) at small squeeze factor we have

$$y = r^{1/2}. \quad (II.31)$$

Rewriting the expression (II.30) for the entanglement coefficient in case two-mode gluon states as we can write the condition of the entanglement with taking into account (II.29) and (II.31)

$$0 < \left|t\frac{g^2}{4\sqrt{2}\,k_0}(f_{ahb}f_{ahc}+f_{agb}f_{agc}+f_{ahb}f_{agc}+f_{agb}f_{ahc})\sum_{j\neq l}|\alpha_l^b||\alpha_j^c|\sin(\gamma_j^b+\gamma_j^c)\right| < 1. \quad (II.32)$$

Obviously squeezed gluon states are simultaneously entangled if the amplitudes of the initial gluon coherent fields are small enough. Thus, by analogy with quantum optics we have obtained the two-mode squeezed gluon states which are also entangled as a result of the four-gluon self-interaction. We have proved theoretically the possibility of existence of the gluon single-mode SS at nonperturbative stage of the QCD jet evolution. As one of identification criterion of existence of such gluon states can served correlation function. On the base of LPHD we have analyzed the behaviour of angular correlations of the pions in the hadron jet narrow ring and have compared our results with the corresponding correlation function for photon squeezed states, which was comprehensively investigated in quantum optics. Thus the nonperturbative contribution of the gluon squeezed states to the pion correlation functions has been estimated. Antibunching and bunching of the pions was revealed. QCD evolution leads both to squeezing and entanglement of gluons.Two-mode gluon states with two different colours can lead to $q\bar{q}$ -entangled states role of which could be very significant for explanation of the hadronization and confinement phenomenon.

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III. Strong QCD instantons

A possibility of strong growth of the cross-section of the instanton transitions in high energy collisions was mentioned first for electroweak theory [III.1]. Shortly after this it was shown [III.2] that QCD-instantons can appear as a new channel of deep inelastic scattering andbe observed at the present-day experiments unlike electroweak instantons.QCD instantons provide quantum tunneling between QCDvacuapotencial energy wells with different Chern-Saimon numbers N_{cs}

$$N_{cs} = \frac{g^2}{16\pi^2}\int d^3x\varepsilon_{ijk}\left(A_i^a\partial_j A_k^a + \frac{g}{3}\varepsilon^{abc}A_i^a A_j^b A_k^c\right). \tag{III.1}$$

QCD-instantons can be produced in quark-gluon subprocess. A set of important features of the process (large number of secondary particles, specificbehavior of cross-section and structure functions, large transversal energy flow and others)was already discussed by Schrempp, Ringwald et al [III.3, III.4]. Correlation properties of instanton-induced processes can be considered as new criterionsof the QCD-instanton identification in addition to criterions of Schrempp, Ringwald. Two-particle correlation function [III.5], factorial and Hq-moments [III.6]) showed that instantoninduced processes are characterized by specific form of correlation characteristics at partonlevel. Footprints of these features persist after hadronization [III.10]. In particular normalizedfactorial moments for instanton processes grow very slowly, Hq-moments are characterizedby first minima at $q = 2$ unlike ordinary DIS [III.7, III.8]. Hadronization was takeninto account by means of Monte-Carlo package QCDINS [III.3, III.4, III.10] (the program which generatesQCD-instanton-inducedevents). Usually it is supposed that only minimal number of quarks is produced after "decay" ofinstanton (number of final gluons is supposed to be arbitrary): $q + g \rightarrow (2n_f - 1)q + ngg$. This supposition was used by authors of the package QCDINS [III.3, III.4] as well as in [III.7].Let us we consider instanton-induced processes with arbitrary number of quarks. Contributionof these processes is determined by nonzero fermion propagator [III.9]. After calculation we obtain Poisson distribution on number of quark pairs (for light quarks), which are produced in the instanton processes.Bjorken variable of instanton subprocess $x' > 0.5$. Average number of quark pairs for small $\mathfrak{z}$ ($\mathfrak{z} = (1 - x')/x'$) reads $< n >\sim 3(1 + \mathfrak{z}^2)$, Bjorken variable of instanton subprocess $x' > 0.5$. Contribution of non-zero modes [III.9] can lead to another behavior of characteristics of instantonprocesses and be important for the experimental search of QCD-instantons [III.11].

Thus, key differences of QCD-Instanton induced final states in comparison to perturbative QCD final states are flavour democracy, isotropic decay, high average multiplicity, Poisson distribution of gluons and also specific behaviour of correlation moments.

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IV. Chaos assisted instanton tunneling

It was discovered recently that very important in different branches of science, including nuclear processes of fission and fusion, phenomenon of quantum tunneling, can be accelerated [IV.1] or slowing down [IV.2] up to several orders of magnitude by the small perturbation leading to chaos in classical case. Approach based on instanton technique [IV.3] and namely on chaotic instantonapproach [IV.3, IV.4] gives an analytical prediction for the influence of theperturbation on quantum properties of nonlinear systems . We discuss here the method on a simple quantum mechanical example with the Hamiltonian:

$$\overline{H} = \frac{1}{2}\overline{p}^2 + \omega_0^2 \cos x - \varepsilon x \sum_{n=-\infty}^{+\infty} \delta\left(t - n\overline{T}\right).$$

The systems with spatially periodic potential are well-studied in solid-state physics [IV.5] and instanton physics [IV.6]. Perturbationusedhere was widely exploited in the systems exhibiting quantum chaos [IV.7]. Chaotic instanton is the solution of the Euclidean equations of motion of the perturbed system [IV.3, IV.4]. This configuration is responsible for the enhancement of tunneling. Dynamical tunneling amplitude with the contribution of the chaotic instanton solutions is [IV.3, IV.4]

$$A = N\int_0^{\Delta H} d\xi \int_{-\infty}^{\infty} dc_0 \sqrt{S\left[x^{inst}(\tau,\xi)\right]}\,\exp\left(-S\left[x^{inst}(\tau,\xi)\right]\right) \approx N\sqrt{8\omega_0}\,\Gamma e^{-8\omega_0}\exp\left(\frac{\pi\Delta H}{\omega_0}\right).$$

It has exponentially enhancing factor $A = A_0 \exp\left(\frac{\pi\Delta H}{\omega_0}\right)$, where A_0 is tunneling amplitude for the Hamiltonian without perturbation. Thus, small perturbation leading to chaos can essentially enhance the tunnelling rate in comparison with non-perturbed system. Theory of strong interactions (QCD) is the most interesting example, for the method where the investigation of instanton gas (or instanton liquid) could shed light on the structure of hadrons and in applied sphere on details of thermonuclear fusion.

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V. Fractal properties of hadron clusters in Hagedorn bootstrap model

It is demonstrated that widely known Hagedorn statistical bootstrap model in the framework of which for the first time conception of phase transition critical temperature in quark gluon plasma was introduced describes intermittent behavior (fractal dimension) in high energy ion-ion collisions [V.1–V.7].

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Conclusion. By analytical and numerical calculations it was obtained that:

I. Interaction of colour particles during evolution with stochastic QCD vacuum considered as environment leads to the decoherence,mixed quantum states, instability of motion, loss (confinement) of colour (decreasing of information of colour, Von Neumann entropy, purity, fidelity), to order to chaos transition.

II. Nonperturbative interaction of quarks and gluons through four gluon self-interaction part of QCD Lagrangian (in particular in jets at nonperturbaive stage of jet evolution) leads to squeezed and entangled states of gluons, quarks (under interaction of gluons with different colours) and hadrons (under condition of local parton-hadron duality), thus not only for electromagnetic but also for strong interactions. As one of identification criteria of existence of squeezed gluon states can serve correlation function behaviour of angular and rapidity correlations of the pions (antibunching and bunching) in the hadron jet narrow ring.

III. Strong (QCD) instantons which provide quantum tunneling between QCD vacua of potential energy wells with different Chern-Saimon numbers N_{cs} except usual key differences (flavour democracy, chirality violation, isotropic decay) have specific correlation properties : high average multiplicity, Poisson distribution of gluons (hadrons), very slow growth of normalized factorial moments and Hq-moments characterized by first minima at $q = 2$ unlike ordinary case (at $q = 6$).

IV. Chaotic instantons are introduced and discussed being the solutions of the Euclidean equations of motion of the periodic perturbed system, leading to chaos in classical case and provide accelerated or delayed quantum tunneling and are very useful for analysis in particular of nuclear processes of fission and fusion.

V. It is demonstrated that widely known Hagedorn statistical bootstrap model in the framework of which for the first time conception of critical phase transition temperature in quark gluon plasma was introduced describes intermittent behavior (fractal dimension) in high energy ion-ion collisions.

Information about the authors

Viatcheslav I. Kuvshinov – D. Sc. (Physics and Mathematics), Professor, Head of the Laboratory for Problems of Nuclear Physics and Safety, Joint Institute for Power and Nuclear Research – Sosny of the National Academy of Sciences of Belarus (P.O. box 119, 220109, Minsk, Republic of Belarus). E-mail: v.kuvshinov@sosny.bas-net.by

Информация об авторах

Кувшинов Вячеслав Иванович – доктор физико-математических наук, профессор, заведующий лабораторией проблем ядерной физики и безопасности, Объединенный институт энергетических и ядерных исследований – Сосны Национальной академии наук Беларуси (а/я 119, 220109, г. Минск, Республика Беларусь). E-mail: v.kuvshinov@sosny.bas-net.by

Evgeny G. Bagashov – M. Sc. (Physics and Mathematics), Junior Researcher of the Joint Institute for Power and Nuclear Research – Sosny of the National Academy of Sciences of Belarus (P.O. box 119, 220109, Minsk, Republic of Belarus). E-mail: bagashov@sosny.bas-net.by

Valerii A. Shaporov – Senior Researcher of the Joint Institute for Power and Nuclear Research – Sosny of the National Academy of Sciences of Belarus (P.O. box 119, 220109, Minsk, Republic of Belarus). E-mail: shaparau@sosny.bas-net.by

Andrei V. Kuzmin – Ph. D. (Physics and Mathematics), Director General, Joint Institute for Power and Nuclear Research – Sosny of the National Academy of Sciences of Belarus (P.O. box 119, 220109, Minsk, Republic of Belarus). E-mail: avkuzmin@sosny.bas-net.by

Багашов Евгений Геннадьевич – магистр, младший научный сотрудник лаборатории проблем ядерной физики и безопасности, Объединенный институт энергетических и ядерных исследований – Сосны Национальной академии наук Беларуси (а/я 119, 220109, г. Минск, Республика Беларусь). E-mail: bagashov@sosny.bas-net.by

Шапоров Валерий Александрович – старший научный сотрудник лаборатории проблем ядерной физики и безопасности, Объединенный институт энергетических и ядерных исследований – Сосны Национальной академии наук Беларуси (а/я 119, 220109, г. Минск, Республика Беларусь). E-mail: shaparau@sosny.bas-net.by

Кузьмин Андрей Владимирович – кандидат физико-математических наук, генеральный директор Объединенного института энергетических и ядерных исследований Сосны – Национальной академии наук Беларуси (а/я 119, 220109, г. Минск, Республика Беларусь). E-mail: avkuzmin@sosny.bas-net.by