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**A. J. Silenko**

*Institute for Nuclear Problems of the Belarusian State University, Minsk, Belarus*
*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna, Russia*

# QUASI-MAGNETIC RESONANCE IN STORAGE RING ELECTRIC-DIPOLE-MOMENT EXPERIMENTS

**Abstract.** A general theoretical description of a magnetic resonance is presented. This description is necessary for a detailed analysis of spin dynamics in electric-dipole-moment experiments in storage rings. General formulas describing a behavior of all components of the polarization vector at magnetic resonance are obtained for arbitrary initial polarization. Quasi-magnetic resonances for particles and nuclei moving in non-continuous perturbing fields of accelerators and storage rings are considered. Distinguishing features of quasi-magnetic resonances in storage ring electric-dipole-moment experiments are investigated. The formulas for the effect caused by the electric dipole moment are derived. Main systematical errors are discussed.

**Keywords:** spin, magnetic resonance, electric-dipole moment

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**А. Я. Силенко**

*Институт ядерных проблем Белорусского государственного университета, Минск, Беларусь*
*Лаборатория теоретической физики им. Н. Н. Боголюбова,*
*Объединенный институт ядерных исследований, Дубна, Россия*

## КВАЗИМАГНИТНЫЙ РЕЗОНАНС В ЭКСПЕРИМЕНТАХ ПО ПОИСКУ ЭЛЕКТРИЧЕСКИХ ДИПОЛЬНЫХ МОМЕНТОВ В НАКОПИТЕЛЬНЫХ КОЛЬЦАХ

**Аннотация.** Представлено общее теоретическое описание магнитного резонанса, которое необходимо для детального анализа динамики спина в экспериментах по поиску электрических дипольных моментов в накопительных кольцах. Для произвольной начальной поляризации получены общие формулы, описывающие эволюцию всех компонент вектора поляризации при магнитном резонансе. Рассмотрен квазимагнитный резонанс для частиц и ядер, движущихся в прерывных возмущающих полях ускорителей и накопительных колец. Исследованы отличительные черты квазимагнитных резонансов в экспериментах по поиску электрических дипольных моментов в накопительных кольцах. Выведены формулы для эффекта, обусловленного электрическим дипольным моментом. Обсуждены основные систематические ошибки.

**Ключевые слова**: спин, магнитный резонанс, электрический дипольный момент

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**Introduction.** The magnetic resonance (MR) is a powerful tool of investigation of basic properties of particles and nuclei. The MR is also successfully used for studies of atoms in condensed matters. The theory of the MR is presented in many books (see, e.g., [1, 2]) and research articles. As a rule, the MR is a spin resonance and its classical and quantum-mechanical descriptions are equivalent. In the present work, we rigorously describe spin dynamics at the MR and apply the general theory for an analysis of resonance effects in electric-dipole-moment experiments with polarized beams in storage rings.

The use of the MR in nuclear, particle, atomic, and condensed matter physics consists in a determination of a spin deflection from the initial vertical direction. To find the magnetic moment, one measures dynamics of the vertical polarization. An exhaustive analysis of the spin evolution in storage ring electric-dipole-moment (EDM) experiments needs an advanced description of magnetic and quasimagnetic resonances. In this case, spin interactions of moving particles and nuclei with magnetic and electric fields are defined by the Thomas-Bargmann-Mishel-Telegdi (T-BMT) equation [3] or by its extension taking into account the EDM [4, 5, 6]. An investigation of quasimagnetic resonances caused by the EDM is important for planned experiments with a rf electric-field flipper and a rf Wien filter (see [7, 8, 9]). A change of the spin (pseudo)vector is orthogonal to the spin direction. One needs therefore to measure minor (horizontal) polarization components when a resonance is stimulated by a comparatively weak interaction. In particular, this situation takes place for a search for EDMs. In storage ring experiments, it can be convenient to use an initial horizontal beam polarization and to measure an evolution of the vertical spin component. A needed experimental precision is very high. In addition, the resonance fields of the rf electric-field flipper and the rf Wien filter are noncontinuous. For these reasons, general formulas describing spin dynamics at the magnetic and quasimagnetic resonances and their specific application are necessary.

The system of units $\hbar = 1, c = 1$ is used. We include $\hbar$ and $c$ into some equations when this inclusion clarifies the problem.

**1. General classical description of nuclear magnetic resonance.** In this section, we consider a usual design of the MR and obtain general equations describing the spin dynamics. While the results obtained are mostly known, the presented study allows us to apply a common approach for a consideration of classical and quantum-mechanical effects.

Let a spinning nucleus be placed into the magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$ and the angular frequency of the spin rotation is equal to $\omega_0$. In this case, a rotating or oscillating horizontal magnetic field with a closed angular frequency $\omega \approx \omega_0$ can significantly deflect the nucleus spin from the initial vertical direction. In particular, this effect allows one to measure magnetic moments of nuclei/particles with a high precision.

As a rule, one applies the main vertical magnetic field $\mathbf{B}_0$ and the oscillating horizontal magnetic field $\boldsymbol{\mathcal{B}}\cos(\omega t + \chi)$. In this case, the spin-dependent part of the classical Hamiltonian is given by

$$H = \boldsymbol{\omega}_0 \cdot \boldsymbol{\zeta} + 2\boldsymbol{\mathcal{E}} \cdot \boldsymbol{\zeta}\cos(\omega t + \chi), \quad \boldsymbol{\omega}_0 = -\frac{g_N \mu_N}{\hbar}\mathbf{B}_0, \\ \boldsymbol{\mathcal{E}} = -\frac{g_N \mu_N}{2\hbar}\boldsymbol{\mathcal{B}}, \tag{1}$$

where $\boldsymbol{\omega}_0$ is the angular velocity of the spin precession at the absence of the horizontal magnetic field, $\boldsymbol{\zeta}$ is the spin (pseudo)vector, $g_N$ is the nuclear $g$-factor, and $\mu_N$ is the nuclear magneton. For particles, $g_N\mu_N$ should be replaced with $eg\hbar/(2m)$, where $g = 2mc\mu/(es)$.

The direction of the (pseudo)vector $\boldsymbol{\omega}_0$ defines the orientation of the so-called stable spin axis. In the absence of oscillating fields, the spin remains stable if it is initially aligned along this direction. If the initial spin orientation is different, the spin describes a cone around the direction of $\boldsymbol{\omega}_0$. The stable spin axis is a static quantity defined *before* activating the rf.

It is preferable to decompose the oscillating horizontal magnetic field into two magnetic fields rotating in opposite directions. The amplitudes of the rotating magnetic fields are equal to $\mathcal{B}/2$. We suppose that $\boldsymbol{\omega}$ is close to $\boldsymbol{\omega}_0$. In this case, one rotating field is resonant and an effect of another rotating field can be neglected. Let us direct $\boldsymbol{\mathcal{E}}$ along the $x$ axis:

$$\boldsymbol{\mathcal{E}} = \mathcal{E}\mathbf{e}_x. \tag{2}$$

This direction is not important in the considered case. The turn of the direction of $\boldsymbol{\mathcal{E}}$ by the angle $\varphi$ is equivalent to the change of phase of the rotating resonant field by the same angle.

To calculate the spin dynamics, it is convenient to use the frame rotating about the $z$ axis with the angular velocity $\boldsymbol{\omega}$. We suppose that the direction of the frame rotation coincides with the direction of the spin rotation, $\boldsymbol{\omega}_0$. The horizontal magnetic field rotating in the lab frame becomes constant in the rotating frame. In this frame, the spin rotates about the $z$ axis with the angular frequency $\omega_0 - \omega$ and the total angular velocity of the spin rotation is equal to

$$\mathbf{\Omega} = \boldsymbol{\omega}_0 - \boldsymbol{\omega} + \boldsymbol{\mathcal{E}}, \quad \Omega = \sqrt{(\omega_0 - \omega)^2 + \mathcal{E}^2}. \tag{3}$$

We disregard the rotating field which rotation direction is opposite to that of the spin. The vectors $\boldsymbol{\omega}_0$ and $\boldsymbol{\omega}$ are parallel and $\boldsymbol{\mathcal{E}}$ is the vector with the constant module $\mathcal{E}$ rotating in the horizontal plane.

We should determine an evolution of the polarization vector **P**. When the initial spin direction is defined by the spherical angles θ and ψ,

$$P_x(0) = \sin\theta\cos\psi, \quad P_y(0) = \sin\theta\sin\psi, \quad P_z(0) = \cos\theta, \tag{4}$$

the final result is given by [10]

$$\begin{gathered}
P_x(t) = \cos\Omega t\sin\theta\cos(\omega t + \psi) + \\
+\frac{\mathcal{E}^2}{\Omega^2}(1-\cos\Omega t)\sin\theta\cos(\psi-\chi)\cos(\omega t+\chi) - \frac{\omega_0-\omega}{\Omega}\sin\Omega t\sin\theta\sin(\omega t+\psi) + \\
+\frac{\mathcal{E}}{\Omega}\left[\frac{\omega_0-\omega}{\Omega}\left(1-\cos\Omega t\right)\cos(\omega t+\chi) + \sin\Omega t\sin(\omega t+\chi)\right]\cos\theta, \\
P_y(t) = \frac{\omega_0-\omega}{\Omega}\sin\Omega t\sin\theta\cos(\omega t+\psi) + \cos\Omega t\sin\theta\sin(\omega t+\psi) + \\
+\frac{\mathcal{E}^2}{\Omega^2}\left(1-\cos\Omega t\right)\sin\theta\cos(\psi-\chi)\sin(\omega t+\chi) + \\
+\frac{\mathcal{E}}{\Omega}\left[\frac{\omega_0-\omega}{\Omega}\left(1-\cos\Omega t\right)\sin(\omega t+\chi) - \sin\Omega t\cos(\omega t+\chi)\right]\cos\theta, \\
P_z(t) = \frac{(\omega_0-\omega)\mathcal{E}}{\Omega^2}\left(1-\cos\Omega t\right)\sin\theta\cos(\psi-\chi) + \\
+\frac{\mathcal{E}}{\Omega}\sin\Omega t\sin\theta\sin(\psi-\chi) + \left[1-\frac{\mathcal{E}^2}{\Omega^2}\left(1-\cos\Omega t\right)\right]\cos\theta.
\end{gathered} \tag{5}$$

Equation (5) presents the general classical description of spin dynamics at magnetic and quasimagnetic resonances and allows us to conclude that one can use both vertical and horizontal initial polarizations. When the terms proportional to $\mathcal{E}^2$ are neglected, Eq. (5) takes the form

$$\begin{gathered}
P_x(t) = \sin\theta\cos(\omega_0 t+\psi) + \frac{\mathcal{E}}{\omega_0-\omega}\{\cos(\omega t+\chi)\left[1-\cos(\omega_0-\omega)t\right] + \\
+\sin(\omega t+\chi)\sin(\omega_0-\omega)t\}\cos\theta, \\
P_y(t) = \sin\theta\sin(\omega_0 t+\psi) + \frac{\mathcal{E}}{\omega_0-\omega}\{\sin(\omega t+\chi)\left[1-\cos(\omega_0-\omega)t\right] - \\
-\cos(\omega t+\chi)\sin(\omega_0-\omega)t\}\cos\theta, \\
P_z(t) = \cos\theta + \frac{\mathcal{E}}{\omega_0-\omega}\{\left[1-\cos(\omega_0-\omega)t\right]\cos(\psi-\chi) + \\
+\sin(\omega_0-\omega)t\sin(\psi-\chi)\}\sin\theta.
\end{gathered} \tag{6}$$

**2. Magnetic and quasimagnetic resonances for moving particles and nuclei.** Magnetic and quasimagnetic resonances for moving particles and nuclei have some distinguishing features. The main difference from the MR for nuclei at rest is the use of the T-BMT equation [3] or its extension taking into account the EDM [4–6] for a description of spin coupling with external fields. The general equation extended on the EDM defines the angular velocity of spin precession in external electric and magnetic fields in the Cartesian coordinates and has the form [4–6].

$$\frac{d\boldsymbol{\zeta}}{dt}=(\boldsymbol{\Omega}_{T-BMT}+\boldsymbol{\Omega}_{EDM})\times\boldsymbol{\zeta},$$

$$\boldsymbol{\Omega}_{T-BMT}=\frac{e}{m}\left[\left(G+\frac{1}{\gamma+1}\right)\boldsymbol{\beta}\times\mathbf{E}-\left(G+\frac{1}{\gamma}\right)\mathbf{B}+\frac{G\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{B})\boldsymbol{\beta}\right], \tag{7}$$

$$\boldsymbol{\Omega}_{EDM}=-\frac{e\eta}{2m}\left[\mathbf{E}-\frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\mathbf{E})\boldsymbol{\beta}+\boldsymbol{\beta}\times\mathbf{B}\right],\quad \boldsymbol{\beta}=\frac{\mathbf{v}}{c},$$

where $G=(g-2)/2,\ \eta=2mcd/(es),$ and $d$ is the EDM.

Equation (7) is useful when the fields have definite directions relative to the Cartesian coordinates. As a rule, it is not the case for particles and nuclei in accelerators and storage rings. Their motion is cyclic and the fields are usually orthogonal to the beam trajectory. Therefore, it is natural to define the fields and the spin motion relative to the radial and longitudinal coordinates. The use of the cylindrical coordinate system [11] decreases the angular frequency of the spin rotation about the vertical axis by the cyclotron frequency $\omega_c$. It is important that the angular frequencies of the spin rotation about the two horizontal axes remain the same. The resulting angular velocity of the spin rotation in the cylindrical coordinate system is equal to [11]

$$\boldsymbol{\Omega}^{(cyl)}=-\frac{e}{m}\left\{G\mathbf{B}-\frac{G\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{B})+\left(\frac{1}{\gamma^2-1}-G\right)(\boldsymbol{\beta}\times\mathbf{E})+\frac{1}{\gamma}\left[\mathbf{B}_{\|}-\frac{1}{\beta^2}(\boldsymbol{\beta}\times\mathbf{E})_{\|}\right]+\right.$$
$$\left.+\frac{\eta}{2}\left(\mathbf{E}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{E})+\boldsymbol{\beta}\times\mathbf{B}\right)\right\}. \tag{8}$$

The sign $\|$ means a horizontal projection for any vector.

As a rule, the vertical and horizontal components of the magnetic and quasimagnetic fields, **B** and $\boldsymbol{\beta}\times\mathbf{E}$, enter into the expressions for $\boldsymbol{\omega}_0$ and $\boldsymbol{\mathcal{E}}$ with different factors. In particular, this situation takes place for a beam in a purely magnetic storage ring:

$$\boldsymbol{\omega}_0=-\frac{eG}{m}\mathbf{B}_0,\quad \boldsymbol{\mathcal{E}}=-\frac{e}{2m}\left(\frac{1}{\gamma}+G\right)\boldsymbol{\mathcal{B}}. \tag{9}$$

Here $\boldsymbol{\mathcal{B}}$ is the amplitude of the perturbing oscillatory magnetic field. In the purely magnetic storage ring, the *average* radial magnetic field is equal to zero. In a storage ring with electric focusing, it should be counterbalanced by a focusing vertical electric field. In this case, the horizontal components of the average Lorentz force vanish:

$$<\mathbf{F}_{L\|}>=e\left[<\mathbf{E}_{\|}>+<(\boldsymbol{\beta}\times\mathbf{B})_{\|}>\right]=0.$$

To describe spin dynamics in accelerators and storage rings, one often uses the Frenet-Serret (FS) coordinate system. The axes of the FS coordinate system depend on the particle trajectory. Three axes of this coordinate system are directed parallel to the velocity and momentum, parallel to the acceleration vector, and along the binormal orthogonal to these two axes. Relative to the Cartesian coordinate system, the FS one rotates about all three axes, not only around the vertical axis as the cylindrical coordinate system.

To find the angular velocity of the spin motion in the FS coordinate system, it is necessary to subtract an angular velocity of rotation of the vector $\mathbf{N}=\mathbf{p}/p=\mathbf{v}/v$ from $\boldsymbol{\Omega}_{T-BMT}+\boldsymbol{\Omega}_{EDM}$. The angular velocity of the spin motion is equal to [5]

$$\boldsymbol{\Omega}^{(FS)}=-\frac{e}{m}\left[G\boldsymbol{\Omega}-\frac{G\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{\Omega})+\left(\frac{1}{\gamma^2-1}-G\right)(\boldsymbol{\beta}\times\mathbf{E})+\frac{\eta}{2}\left(\mathbf{E}-\frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\mathbf{E})+\boldsymbol{\beta}\times\mathbf{B}\right)\right]. \tag{10}$$

A comparison of the cylindrical and FS coordinate systems has been made in Ref. [12].

A detailed consideration of evolution of all spin components is necessary because an interaction stimulating a resonance can be very weak. Evidently, $d\boldsymbol{\zeta}/(dt)\perp\boldsymbol{\zeta}$. When the initial beam polarization is vertical, one needs to measure horizontal spin components. When the initial beam polarization is horizontal, it is convenient to monitor the vertical spin component. These two possibilities may be realized in storage ring experiments on a search for EDMs [13–15]. In these cases, the action of oscillating fields on the EDM stimulates a resonance while their action on the magnetic moment does not bring any resonance effects. This takes place because the quantities $\boldsymbol{\Omega}_{T-BMT}$ and $\boldsymbol{\Omega}_{EDM}$ in Eq. (7) are usually orthogonal. The corresponding quantities in Eqs. (8) and (10) possess the same property. In any case, spin resonances originated from the EDM are quasimagnetic and are not magnetic. We can also mention that the case when the $x$ and $z$ components of the angular velocity of spin precession are constant corresponds to the conditions of the EDM experiment based on the frozen spin method [16].

It has been proven in [15] that the use of the initial vertical polarization cancels some systematical errors. The use of the initial horizontal polarization does not lead to such a cancellation. However, the initial vertical polarization can meet other problems [15].

We can conclude that specific conditions of the magnetic and quasimagnetic resonances for particles and nuclei moving in accelerators and storage rings influence only parameters $\omega_0$ and $\mathcal{E}$ but do not change the general equation (5) defining the spin dynamics. A calculation of small corrections appearing in exact solutions needs a modification of initial equations. This problem has been considered in [10].

It can be added that an extremely high precision of storage ring EDM experiments needs taking into account tensor electric and magnetic polarizabilities for nuclei with spin $s\geq 1$ (e.g., deuteron) [17]. The tensor magnetic polarizability, $\beta_T$, produces the spin rotation with two frequencies instead of one, beating with a frequency proportional to $\beta_T$, and causes transitions between vector and tensor polarizations [17–19]. A beam with an initial tensor polarization acquires a final vector polarization [14, 19, 20]. Resonance effects caused by the tensor polarizabilities have been calculated in [17, 18, 20]. A comparison of spin dynamics conditioned by the tensor polarizabilities and the EDM has been carried out in [14, 20, 21].

**3. Quasimagnetic resonance in a noncontinuous perturbing field.** The next problem which should be taken into consideration in connection with the storage ring EDM experiments is a discontinuity of perturbing fields. In the planned EDM experiments with protons, deuterons, and $^3$He ions at COSY [13, 22], one will use resonance stimulations with a rf electric-field flipper and a rf Wien filter. The both devices create oscillatory fields. The frequencies of the perturbing fields are synchronized with that of the spin frequency. The both devices provide for standard conditions of the MR. Semertzidis [23] and Nikolaev [24] have compared the actions of the rf electric-field flipper and the rf Wien filter on the spin. Orlov has shown [26] that a part of the longitudinal spin component is frozen (constant in time) in the oscillatory radial electric field. A more advanced theoretical analysis has been fulfilled in Ref. [7]. The theoretical calculations agree with spin tracking [7, 23].

The rf Wien filter unlike the rf electric-field flipper does not affect the motion of particles and nuclei. This is a great advantage of the former device. The latter device can be used only if it does not destroy the beam stability.

JEDI collaboration plans to perform main experiments with the rf Wien filter [8, 9, 22]. The static version of this filter is frequently used to turn the spin without an effect on beam dynamics. The rf electric-field flipper may be applied in precursor experiments [13]. The initial beam polarization is planned to be vertical. The initial horizontal beam polarization can also be used.

The stimulating frequency, $\omega'$, should either (almost) coincide with that of the spin rotation, $\omega_0$, or differ by $n\omega_c$ $(n=\pm1,\pm2,...)$, where $\omega_c$ is the cyclotron frequency. This property can be properly substantiated and a rigorous quantitative description of the spin evolution can be given.

One usually puts devices like the electric-field flipper and the rf Wien filter into a straight section of the storage ring. Since lengths of the flipper and the filter are small as compared with the ring circumference, an approximation of the perturbing field by the delta function is permissible. An expansion of the delta function into the Fourier series is defined by the known formula:

$$\sum_{n=-\infty}^{\infty}\delta(\Phi-2\pi n)=\frac{1}{2\pi}+\frac{1}{\pi}\sum_{n=1}^{\infty}\cos(n\Phi)=\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\cos(n\Phi),$$

where $\Phi=\omega_c t$ is the phase. As a result, the following relations are valid (see, e.g., [25]):

$$\begin{aligned}\sin(\omega' t+\chi)\sum_{n=-\infty}^{\infty}\delta(\Phi-2\pi n)&=\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\sin[(\omega'+n\omega_c)t+\chi]=\\&=\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\sin[(n+\nu)\Phi+\chi],\\\cos(\omega' t+\chi)\sum_{n=-\infty}^{\infty}\delta(\Phi-2\pi n)&=\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\cos[(\omega'+n\omega_c)t+\chi]=\\&=\frac{1}{2\pi}\sum_{n=-\infty}^{\infty}\cos[(n+\nu)\Phi+\chi],\end{aligned}\tag{11}$$

where $\nu=\omega'/\omega_c$ is the modulation tune. In this case, $\omega_0=(n+\nu)\omega_c$ and $\omega'=(\nu_s+K)\omega_c$, where $\nu_s=\omega_0/\omega_c$ is the spin tune.

Equation (11) shows a possibility to use resonance devices at different frequencies. In particular, an appropriate choice for the proton and the deuteron is $K=-2,-3$ and $K=+1,+2$, respectively.

More adequately, the electric fields of the flipper and the filter can be characterized as follows:

$$\begin{gathered}\boldsymbol{\Omega}_{\|}=2\mathcal{E}\cos(\omega' t+\chi),\qquad \mathcal{E}=-\frac{e\eta}{4m}\mathbf{E}(\Phi),\\\mathbf{E}(\Phi)=\begin{cases}\mathbf{E}_0 & \text{if}\quad \Phi\in\left[-\dfrac{\pi l}{C}+2\pi n,\ \dfrac{\pi l}{C}+2\pi n\right]\\[2ex] 0 & \text{if}\quad \Phi\notin\left[-\dfrac{\pi l}{C}+2\pi n,\ \dfrac{\pi l}{C}+2\pi n\right]\end{cases},\quad n=0,\pm1,\pm2,\ldots,\end{gathered}\tag{12}$$

where $l$ is the length of the flipper/filter, $C$ is the ring circumference. The spin-dependent part of the classical Hamiltonian is defined by Eq. (1) and the electric field $\mathbf{E}_0$ is directed radially.

In this case, an expansion into the Fourier series has the form

$$\mathbf{E}(\Phi)=\mathbf{E}_0\sum_{n=-\infty}^{\infty}a_n\cos(n\Phi),\tag{13}$$

where

$$a_0=\frac{l}{C},\qquad a_n=\frac{1}{\pi n}\sin\frac{\pi n l}{C}.\tag{14}$$

As a result,

$$\begin{aligned}\boldsymbol{\Omega}_{\|}&=-\frac{e\eta}{2m}\mathbf{E}_0\sum_{n=-\infty}^{\infty}a_n\cos[(\omega'+n\omega_c)t+\chi]=\\&=-\frac{e\eta}{2m}\mathbf{E}_0\sum_{n=-\infty}^{\infty}a_n\cos[(n+\nu)\Phi+\chi].\end{aligned}\tag{15}$$

Equations (14) and (15) show that the considered devices are not effective for oscillation modes $n>C/(2l)$. Otherwise, the Fourier coefficients for the delta function and the flipper/filter of a finite length agree on condition that $(nl/C)\ll 1$. For a more precise Fourier expansion, one can use real parameters of the resonator fields.

The horizontal component of the angular velocity of the spin precession conditioned by a resonance interaction *of the EDM* with the radial electric field can be presented in the form

$$\mathbf{\Omega}_{\|} = 2\mathcal{E}\cos(\omega t+\chi), \qquad \mathcal{E} = -\frac{e\eta}{4m}a_n\mathbf{E}_0. \tag{16}$$

The resonance frequency, ω, satisfies the condition

$$\omega \equiv \omega' + n\omega_c \approx \omega_0. \tag{17}$$

An expansion of a magnetic field in the rf Wien filter into the Fourier series is very similar. Evidently, a resonance influence of continuous and noncontinuous perturbing fields on the spin is practically the same.

The results presented give an exhaustive description of storage ring resonance effects caused by the magnetic dipole moment (MDM). For this purpose, one should simply substitute needed expressions for $\mathbf{\Omega}_{\|}$ and $\mathcal{E}$ into corresponding equations. Specifically,

$$\mathbf{\Omega}_{\|} = 2\mathcal{E}\cos(\omega t+\chi) = -\frac{e}{m}\left(G+\frac{1}{\gamma}\right)a_n\mathbf{B}_0^{(r)}\cos(\omega t+\chi) \tag{18}$$

for the radial magnetic field and

$$\mathbf{\Omega}_{\|} = 2\mathcal{E}\cos(\omega t+\chi) = -\frac{eg}{2m\gamma}a_n\mathbf{B}_0^{(l)}\cos(\omega t+\chi) \tag{19}$$

for the longitudinal magnetic field. The resonance effect caused by the MDM and stimulated by the rf Wien filter with the radial magnetic and vertical electric fields $\left[(\mathbf{E}_0+\boldsymbol{\beta}\times\mathbf{B}_0^{(r)})_{\|}=0\right]$ is defined by

$$\mathbf{\Omega}_{\|} = 2\mathcal{E}\cos(\omega t+\chi) = -\frac{eg}{2m\gamma^2}a_n\mathbf{B}_0^{(r)}\cos(\omega t+\chi). \tag{20}$$

**4. Distinguishing features of a quasimagnetic resonance in storage ring electric-dipole-moment experiments.** Main distinguishing features of storage ring EDM experiments are a simultaneous influence of external fields on the electric and magnetic dipole moments and the existence of a resonance effect even when the stimulating torque acting *on the EDM* is equal to zero. The last situation takes place when the resonance in a EDM experiment is stimulated by the rf Wien filter with the vertical magnetic and radial electric fields $\left[(\mathbf{E}_0+\boldsymbol{\beta}\times\mathbf{B}_0^{(osc)})_r=0\right]$. The paradoxical property of the existence of the resonance effect on condition that $\mathbf{\Omega}_{EDM}=0$ has been first discovered by Semertzidis [23] with a computer simulation. The existence of this effect has been confirmed and has been rigorously proven by the subsequent theoretical analysis fulfilled in [7, 24, 26].

A very simple explanation of the distinguishing features of a quasimagnetic resonance in storage ring EDM experiments has been given in [10]. This explanation is valid for any initial polarization of particles or nuclei.

Amazingly, there is not an effect of the resonance field on the EDM and the resonance effect *proportional to the EDM* is ensured by the action of the oscillating fields on the MDM. If we neglect small corrections, we can use the general equations obtained in the precedent sections. In this case,

$$\mathcal{E} = -\frac{e\eta}{4m}\cdot\frac{G+1}{G\gamma^2}a_nE_0. \tag{21}$$

In this approximation, an addition of the oscillating vertical magnetic field does not change the resonance effect. However, taking into account terms of the order of δ demonstrates a difference between the EDM effects caused by the rf electric-field flipper and the rf Wien filter (containing the same rf electric-field flipper) [10]. While the difference is small, it is crucial for the establishment of consent between analytical derivations and computer simulations.

One of key problems in EDM experiments is the problem of systematical errors. Equations (8), (10) show that the vertical electric field and the radial and longitudinal magnetic fields may create a resonance effect imitating the presence of the EDM. This effect can take place due to misalignments and imperfections of the oscillating fields in the rf Wien filter. Similarly directed constant imperfection fields can also exist in the storage ring. However, they do not create any *resonance* effect.

To decrease systematic errors, it is necessary to avoid any dependence of the particle motion on the fields of the rf Wien filter. This means canceling the Lorentz force in both radial and vertical directions [7]. With allowance for Eq. (8) and the relation $(\mathbf{E}_0+\boldsymbol{\beta}\times\mathbf{B}_0^{(osc)})\big|_z=0,$ we obtain the formula $\left(\boldsymbol{\beta}=\beta\mathbf{e}_\varphi\right)$

$$\boldsymbol{\Omega}_{\|}=-\frac{e}{m}\cdot\frac{G+1}{\gamma^2}a_n\mathbf{B}^{(r)},\qquad \mathcal{E}=-\frac{e}{2m}\cdot\frac{G+1}{\gamma^2}a_nB_0^{(r)}. \tag{22}$$

This formula agrees with the result obtained in Ref. [7]. A nonzero value of $B_0^{(r)}$ conditions a systematic error. It is rather difficult to distinguish the EDM signal from this systematic error. One can use the fact that these quantities differently depend on the velocity [7].

**Summary.** In the present paper, a general theoretical description of the MR is given. We have derived the general formulas describing a behavior of all components of the polarization vector at the MR and have considered the case of an arbitrary initial polarization.

Distinguishing features of magnetic and quasimagnetic resonances for particles and nuclei moving in accelerators and storage rings (including resonances caused by the EDM) have been investigated in detail. We have considered the quasimagnetic resonance in a noncontinuous perturbing field. We have also fulfilled a detailed description of a quasimagnetic resonance in storage ring EDM experiments.

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## Information about the author

**Alexander J. Silenko** – D. Sc. (Physics and Mathematics), Leading Researcher, Institute for Nuclear Problems of the Belarusian State University (11, Bobruiskaya Str., 220030, Minsk, Republic of Belarus); Leading Researcher of the Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research (6, Joliot-Curie Str., 141980, Dubna, Moscow Region, Russian Federation). E-mail: alsilenko@mail.r

## Информация об авторе

**Силенко Александр Яковлевич** – доктор физико-математических наук, ведущий научный сотрудник Института ядерных проблем Белорусского государственного университета (11, ул. Бобруйская, 220030, г. Минск, Республика Беларусь); ведущий научный сотрудник лаборатории теоретической физики им. Н. Н. Боголюбова Объединенного института ядерных исследований (ул. Жолио-Кюри, 6, 141980, г. Дубна, Московская обл. Российская Федерация). E-mail: alsilenko@mail.ru