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S. L. Cherkas[1], V. L. Kalashnikov[2]

[1]Institute for Nuclear Problems of the Belarusian State University, Minsk, Belarus
[2]Vienna University of Technology, Vienna, Austria

MATTER CREATION AND PRIMORDIAL CMB SPECTRUM IN THE INFLATIONLESS MILNE-LIKE COSMOLOGIES

Abstract. The primordial spectrum of scalar particle's density perturbations is calculated. On the assumption of spectrum universality, i.e., a mean energy density and a typical value of inhomogeneity can be chosen arbitrarily, the form of the spectrum turns out to be completely defined. It is close to the flat Harrison – Zeldovich spectrum, but with the suppression of low-frequency modes.

Keywords: density perturbations, matter creation, quantum evolution

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С. Л. Черкас[1], В. Л. Калашников[2]

[1]Институт ядерных проблем Белорусского государственного университета, Минск, Беларусь
[2]Венский технический университет, Вена, Австрия

РОЖДЕНИЕ МАТЕРИИ И АНИЗОТРОПИЯ МИКРОВОЛНОВОГО РЕЛИКТОВОГО ИЗЛУЧЕНИЯ В БЕЗЫНФЛЯЦИОННЫХ КОСМОЛОГИЯХ

Аннотация. Рассматривается рождение материи в альтернативных безынфляционных космологиях, в которых масштабный фактор растет линейно со временем. Вычисляется первоначальный спектр неоднородностей плотности родившихся скалярных частиц. Если потребовать универсальность спектра, т. е. чтобы среднюю плотность энергии и характерную величину неоднородности плотности энергии можно было задавать произвольно, то форма спектра оказывается полностью фиксированной. Спектр близок по форме к плоскому спектру Хариссона – Зельдовича, но с подавлением низкочастотных мод.

Ключевые слова: возмущения плотности, рождение материи, начальный спектр

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Milne-like cosmologies again attract an attention lately [1–8]. However, although the original Milne universe [9] is open and empty, the flat universes filled with some exotic matter with the overall equation of state $w=-1/3$ is usually considered. Moreover, there could be a deeper background for these cosmologies, relying on the residual vacuum fluctuations [10].

One of the attractive points for such a model is that it solves the horizon problem without inflation. The most accurate cosmological datum is the anisotropy of the cosmic microwave background. However, the confrontation of the Milne-like cosmologies with the experimental data is far to be complete. One of the problems is the primordial spectrum of the density perturbation. In the absence of the inflation, one needs another model for the primordial density perturbation spectrum. Some steps have been done in this direction [7, 11], but they refer to the methodology of the old inflationary paradigm and do not discuss the origin of matter in the universe. In the inflationary scenario, the matter appears as a result of the inflaton field decay. If there is no inflation, then there is no inflaton field and no matter.

1. Wave packet description of the quantum field in the expanding universe. Quantum fields on the classical background were carefully considered earlier [12]. The main instrument for the description of the quantum field are operators of creation and annihilation used for quantization of the field oscillators. For the expanding universe, it seems, another formalism could be convenient. In fact, in a vicinity of singularity, there are no field oscillators because they do not begin to oscillate yet. From the other hand, it was shown that there exist some finite quantities in the singularity, namely momentums of the dynamical variables despite the infinity of the dynamical variables itself [13]. State of the quantum field can be described in terms of the wave packet over eigenfunctions of these momentums.

Let us begin from the conventional formalism for the scalar field $\phi(\eta,\mathbf{r})$ in the expanding universe described by the Lagrangian

$$L=\frac{1}{2}\int a^2\left(\phi'^2-(\nabla\phi)^2\right)d^3\mathbf{r}, \quad (1)$$

where $a(\eta)$ is the scale factor.

It is suggested that the metric tensor of the universe corresponds to the interval

$$ds^2=a^2(\eta)(d\eta^2-d\mathbf{r}^2),$$

where η is the conformal time. Expanding of scalar field over Fourier series

$$\phi(\mathbf{r})=\sum_{\mathbf{k}}\varphi_{\mathbf{k}}e^{i\mathbf{k}\mathbf{r}} \quad (2)$$

allows rewriting the Lagrangian (1) as

$$L=\frac{a^2}{2}\sum_{\mathbf{k}}\varphi_{\mathbf{k}}'\varphi_{-\mathbf{k}}'-k^2\varphi_{\mathbf{k}}\varphi_{-\mathbf{k}}. \quad (3)$$

Corresponding momentums are $\pi_{\mathbf{k}}=\dfrac{\partial L}{\partial\varphi_{\mathbf{k}}'}=a^2\varphi_{-\mathbf{k}}'$. In the terms of momentums, the Hamiltonian $H=L-\sum_{\mathbf{k}}\varphi_{\mathbf{k}}'\dfrac{\partial L}{\partial\varphi_{\mathbf{k}}'}$ is

$$H=\frac{1}{2}\sum_{\mathbf{k}}\frac{\pi_{\mathbf{k}}\pi_{-\mathbf{k}}}{a^2}+a^2k^2\varphi_{\mathbf{k}}\varphi_{-\mathbf{k}}. \quad (4)$$

Each **k**-mode satisfies the equation of motion

$$\varphi_{\mathbf{k}}''+\frac{2a'}{a}\varphi_{\mathbf{k}}'+k^2\varphi_{\mathbf{k}}=0. \quad (5)$$

Quantization in terms of the creation and annihilation operators consists in postulating [12]

$$\hat{\varphi}_{\mathbf{k}}=\hat{a}_{-\mathbf{k}}^{+}u_k^{*}(\eta)+\hat{a}_{\mathbf{k}}u_k(\eta), \quad (6)$$

where functions u_k satisfy to the equation of motion (5) and the relation

$$a^2(\eta)(u_k u_k'^{*}-u_k^{*}u_k')=i. \quad (7)$$

Let us prove that for a wide class of dependencies $a(\eta)$, such that the kinetic (first) term is dominating over the potential (second) term in Eq. (5) in the vicinity of the singularity, the momentums are finite quantities. Momentum corresponding to the **k**-mode can be written

$$\hat{\pi}_{\mathbf{k}}=a^2\hat{\varphi}_{-\mathbf{k}}'=a^2(\eta)\left(\hat{a}_{-\mathbf{k}}u_k'(\eta)+\hat{a}_{\mathbf{k}}^{+}u_k^{*\prime}(\eta)\right). \quad (8)$$

Near singularity, the function $u_k(\eta)$ satisfies the equation (5) without the last term asymptotically. This equation can be converted into the form

$$\frac{d}{d\eta}\left(a(\eta)^2u_k'(\eta)\right)=0. \quad (9)$$

From the equations (8) and (9) one may conclude that the momentums $\hat{\pi}_{\mathbf{k}}$ are asymptotically some constant operators in the vicinity of singularity. The assumption, that the kinetic term is dominant in the vicinity of singularity, is valid, for instance, for dependencies $a(\eta)\sim\eta^n$. In particular, these dependencies include $a(\eta)\sim\eta$ (radiation background) and $a(\eta)\sim\eta^2$ (matter background). Above assumption is not valid for Milne-like cosmology in which $a(\eta)\sim\exp(\mathcal{H}\eta)$, but in the next section we argue that the Milne-like behavior begins not from the singularity but some later.

Let us define a time-independent operator

$$\hat{P}_{\mathbf{k}}=\alpha_k\hat{a}_{-\mathbf{k}}+\alpha_k^*\hat{a}_{\mathbf{k}}^+, \tag{10}$$

where complex constants are $\alpha_k=a^2(\eta)u_k'(\eta)|_{\eta\to 0}$ and define one more operator

$$\hat{X}_{\mathbf{k}}=b_k\exp(i\theta_k)\hat{a}_{\mathbf{k}}+b_k\exp(-i\theta_k)\hat{a}_{-\mathbf{k}}^+. \tag{11}$$

The requirement that the commutation relations are satisfied

$$[\hat{P}_{\mathbf{k}},\hat{X}_{\mathbf{q}}]=-i\delta_{\mathbf{k},\mathbf{q}}. \tag{12}$$

allows finding the constants b_k. Taking into account that $[\hat{a}_{\mathbf{k}},\hat{a}_{\mathbf{k}}^+]=1$ we come to

$$b_k=-\frac{i}{\alpha_k e^{-i\theta_k}-\alpha^*{}_k e^{i\theta_k}}. \tag{13}$$

With the help of the equations (10), (11) and (13) we can express creation and annihilation operators through $\hat{X}_{\mathbf{k}}$ and $\hat{P}_{\mathbf{k}}$:

$$\hat{a}_{\mathbf{k}}=\frac{\hat{P}_{\mathbf{k}}^+}{\alpha_k-e^{2i\theta_k}\alpha_k^*}-i\alpha_k^*\hat{X}_{\mathbf{k}},\qquad \hat{a}_{-\mathbf{k}}^+=\frac{e^{2i\theta_k}\hat{P}_{\mathbf{k}}^+}{\alpha_k^*e^{2i\theta_k}-\alpha_k}+i\alpha_k\hat{X}_{\mathbf{k}}. \tag{14}$$

Substituting (14) into (6) we come to

$$\hat{\varphi}_{\mathbf{k}}(\eta)=\frac{\hat{P}_{\mathbf{k}}^+\left(u_k(\eta)-e^{2i\theta_k}u_k^*(\eta)\right)}{\alpha_k-\alpha_k^*e^{2i\theta_k}}+i\hat{X}_{\mathbf{k}}\left(\alpha_k u_k^*(\eta)-\alpha_k^*u_k(\eta)\right). \tag{15}$$

Realization of the operators $\hat{P}_{\mathbf{k}}$ and $\hat{X}_{\mathbf{k}}$ is convenient to take in the form $\hat{P}_{\mathbf{k}}=P_{\mathbf{k}}, \hat{X}_{\mathbf{k}}=i\frac{\partial}{\partial P_{\mathbf{k}}}$. The equation (13) allows describing the quantum evolution of the fields using the wave packet $C(P_{\mathbf{q}})$ which is set in the singularity. For instance, mean value of $\hat{\varphi}_{\mathbf{k}}(\eta)$ over a wave packet takes the form

$$\begin{aligned}<\psi|\hat{\varphi}_{\mathbf{k}}(\eta)|\psi>=&\frac{e^{-i\theta_k}u_k(\eta)-e^{i\theta_k}u_k^*(\eta)}{e^{-i\theta_k}\alpha_k-e^{i\theta_k}\alpha_k^*}\int\left(C(P_{\mathbf{q}})\right)^*P_{\mathbf{k}}^*C(P_{\mathbf{q}})\mathcal{D}P_{\mathbf{q}}\mathcal{D}P_{\mathbf{q}}^*-\\&-\left(\alpha_k u_k^*(\eta)-u_k(\eta)\alpha_k^*\right)\int\left(C(P_{\mathbf{q}})\right)^*\frac{\partial}{\partial P_{\mathbf{k}}}C(P_{\mathbf{q}})\mathcal{D}P_{\mathbf{q}}\mathcal{D}P_{\mathbf{q}}^*,\end{aligned} \tag{16}$$

where integration implies $\mathcal{D}P_{\mathbf{k}}=dP_0dP_{\mathbf{k}1}dP_{\mathbf{k}1}^*dP_{\mathbf{k}2}dP_{\mathbf{k}2}^*\ldots$. The integral over $dzdz^*\equiv\frac{\rho d\rho d\phi}{2\pi i}$, $z=\rho e^{i\phi}$ is understood in the holomorphic representation [14]. Let us consider the transformation of the wave packet $C(P_{\mathbf{k}})\to C(P_{\mathbf{k}})\exp\left(-i\sum_{\mathbf{q}}g_qP_{\mathbf{q}}^*P_{\mathbf{q}}\right)$ in Eq. (16), where g_k are some real constants. Under summation on $\mathbf{q}$ one should take into account that $P_{-\mathbf{q}}=P_{\mathbf{q}}^*$. As can see, the transformation is equivalent to the change of the phase θ_k as

$$\theta_k \to \arctan\left(\frac{\tan\theta_k\left(2+ig_k\left(\alpha_k^2-\alpha_k^{*2}\right)\right)-g_k(\alpha_k-\alpha_k^*)^2}{2-g_k\left((\alpha_k+\alpha_k^*)^2\tan\theta_k+i\left(\alpha_k^2-\alpha_k^{*2}\right)\right)}\right). \tag{17}$$

That is the phases θ_k could be considered twofold: mathematically they are the phases of the basis functions $u_k(\eta)$, but physically they are the property of the quantum state under the chosen basis u_k, because they are equivalent to the constants g_k. Further, we shall consider the Gaussian wave packets $C(P_\mathbf{k})=\exp\left(-\sum_\mathbf{q}\Delta_\mathbf{q}P_\mathbf{q}^*P_\mathbf{q}\right)$, with the real constants $\Delta_\mathbf{k}$ and, besides, characterize a quantum state by the phases θ_k. In another word, we may consider θ_k as some phase of the function $u_k(\eta)$, and this phase is a property of the quantum state as well as a width Δ_k of the Gaussian distributions. Let us emphasize, that the matter is encoded in the singularity here. It is not created from the vacuum. Even for massive particle amount of matter created from the vacuum is not sufficient to explain the content of the universe in Milne-like cosmology [15], if to take observed conformal Hubble constant $\mathcal{H}$.

2. Model of the universe expansion. It is evident that the Milne-like behavior is hardly extendable to the vicinity of singularity. Let us consider a heuristic model to describe the way in which the universe could come to the Milne-like expansion. Consider Hamiltonian in which the universe metric is suggested to be uniform, but the non-uniform scalar field presents [10, 16]:

$$H=-\frac{1}{2}M_p^2a'^2+\frac{1}{2}\sum_\mathbf{k}\frac{\pi_\mathbf{k}\pi_\mathbf{k}^+}{a^2}+a^2k^2\phi_\mathbf{k}\phi_\mathbf{k}^+. \tag{18}$$

Initially, the last term corresponding to the potential energy of the field oscillators does not play a role. Consequently, $a(\eta)\sim\sqrt{\eta}$ under the small conformal time, because $a'^2\sim\frac{\pi_\mathbf{k}\pi_\mathbf{k}^*}{a^2}$ and $\pi_\mathbf{k}\sim\text{const}$. When the field oscillators begin to oscillate, the expansion changes from $a(\eta)\sim\sqrt{\eta}$ to $a(\eta)\sim\exp(\mathcal{H}\eta)$, i.e. $a(t)\sim\mathcal{H}t$ in cosmic time [10]. The late-time rate of the universe expansion also differs from the Milne-like, because universe accelerates due to residual vacuum fluctuations [10]. Here, for simplicity, we will not take this acceleration into account.

Due to UV cut off of the comoving momentums, high energy oscillators at the Planck frequencies give the main contribution to the universe expansion [10, 16]. Thus, the change of the expansion rate occurs near the Planck time and can be described by the model

$$a(\eta)=\begin{cases}\gamma\sqrt{\eta}+\beta\eta, & 0<\eta<\eta_1,\\ B\exp(\mathcal{H}\eta), & \eta>\eta_1,\end{cases} \tag{19}$$

where η_1 is of the order of the Planck time. For the smooth splicing of the function and their derivatives, the linear term $\beta\eta$ has been introduced in the Eq. (19). Values of the coefficients are equal to

$$\gamma=\frac{2Be^{\mathcal{H}\eta_1}(1-\mathcal{H}\eta_1)}{\sqrt{\eta_1}}, \qquad \beta=-\frac{Be^{\mathcal{H}\eta_1}(1-2\mathcal{H}\eta_1)}{\eta_1}. \tag{20}$$

Milne-like models seem attractive because it could solve the horizon problem without inflation, but this puts the constraint on the coefficient B in Eqs. (19), (20). One of the formulations of the horizon problem is that the last scattering surface consists of some causally disconnected regions. Let η_0 will be today number of conformal time corresponding to the present-day scale factor a_0. The horizon is the present observable part of the universe which could be reached by the light:

$$R(t_0)=a(t_0)\int_0^{t_0}\frac{dt'}{a(t')}. \tag{21}$$

In comoving coordinates $R(t_0)/a(t_0)$, the horizon is simply $\int_0^{\eta_0} d\eta = \eta_0$, i.e. conformal time from the beginning of the universe, thus, $R(t_0) = a_0\eta_0$. The size of the region corresponding to the present horizon at the time of the last scattering is $R(t_0)\dfrac{a_{LS}}{a_0}$. This size should be an order of or less than the horizon at last scattering time $R(t_{LS})$:

$$R(t_0)\frac{a_{LS}}{a_0} \le R(t_{LS}) = a_{LS}\eta_{LS}, \tag{22}$$

otherwise it will consist of a number of causally disconnected regions. Substituting of $\eta_0 = \dfrac{1}{\mathcal{H}}\ln\dfrac{a_0}{B}$ and $\eta_{LS} = \dfrac{1}{\mathcal{H}}\ln\dfrac{a_{LS}}{B}$ to Eq. (22) gives

$$\ln\frac{a_0}{B} \approx \ln\frac{a_{LS}}{B}, \tag{23}$$

where the sign $\le$ is changed by $\approx$ because the first one is principally impossible. The red shift of the last scattering surface is $z_{LS} \approx 1100$, i.e. $a_0/a_{LS} \approx 10^{-2}$, thus B must be sufficiently small. For instance, taking $a_0 \equiv 1$ and $B = 10^{-30}$ one has $\ln 10^{30} \approx \ln 10^{28}$. The problem of the horizon is solved in the linear cosmologies by the minimal way: although the region at last scattering surface corresponding to the present horizon is less than the horizon at last scattering, they equal approximately if the constant B is sufficiently small.

Solutions of the equations (5) (although this equation is for $\varphi_{\mathbf{k}}$, u_k obeys it, as well) in the different regions can be written as

$$u_k(\eta) = \begin{cases} \dfrac{\sqrt{\pi}}{2\gamma}\left(1 + \dfrac{2i}{\pi}\left(\dfrac{2\gamma}{\gamma+\beta\sqrt{\eta}} - 2\ln\left(\gamma+\beta\sqrt{\eta}\right) + \ln(\eta)\right)\right), & 0 < \eta < \eta_1, \\ \dfrac{e^{-\mathcal{H}\eta - i\eta\sqrt{k^2-\mathcal{H}^2}}}{\sqrt{2}B\sqrt[4]{k^2-\mathcal{H}^2}}, & \eta > \eta_1. \end{cases} \tag{24}$$

Since η_1 is very small compared to the $1/k$ of interest, an approximate non-oscillating solution is taken in the expression (24) at $0 < \eta < \eta_1$. Besides, there is an overall phase in the functions u_k which influences the matter production through the quantities α_k. For calculations, in fact, only last time pieces of the function of $u_k(\eta)$ are needed and, besides, complex quantities α_k discussed in section 1.

2. Mean energy density. Let us calculate a mean energy density of the created particles defined as

$$\bar{\rho} = < C[P] | \hat{\rho} | C[P] > - < 0 | \hat{\rho} | 0 >, \tag{25}$$

where an average vacuum density is extracted and

$$\hat{\rho} = \frac{1}{V}\int_V \left(\frac{\hat{\phi}'^2}{2a^2} + \frac{(\nabla\hat{\phi})^2}{2a^2}\right) d^3\mathbf{r} = \frac{1}{2a^2}\sum_{\mathbf{k}} \hat{\varphi}'_{\mathbf{k}}\hat{\varphi}'_{-\mathbf{k}} + k^2\hat{\varphi}_{\mathbf{k}}\hat{\varphi}_{-\mathbf{k}}, \tag{26}$$

where V is the normalization volume. Substituting the expression (15) into (26), taking into account that $\hat{\varphi}_{-\mathbf{k}} = \hat{\varphi}^{+}_{\mathbf{k}}$ and performing integration over $\mathcal{D}P_{\mathbf{q}}\mathcal{D}P^*_{\mathbf{q}}$ we find

$$\begin{aligned} \bar{\rho} = \frac{1}{a^2}\sum_{|\mathbf{k}|>\mathcal{H}} \frac{1}{4\Delta_k(\alpha_k - \alpha_k^*)^2} \Big(& k^2 u_k(\eta)^2 - 2k^2 u_k(\eta) u_k^*(\eta) + k^2 u_k^*(\eta)^2 + u_k'(\eta)^2 - 2u_k'(\eta) u_k^{*\prime}(\eta) + \\ & + u_k^{\prime *}(\eta)^2 \Big) - \frac{\Delta_k}{4}\Big(\alpha_k^{*2} k^2 u_k(\eta)^2 - 2\alpha_k\alpha_k^* k^2 u_k(\eta) u_k^*(\eta) + \alpha_k^2 k^2 u_k^*(\eta)^2 + \\ & + \left(\alpha_k^* u_k'(\eta) - \alpha_k u_k^{*\prime}(\eta)\right)^2 \Big) - \frac{1}{2}\Big(u_k^{\prime *}(\eta) u_k'(\eta) + k^2 u_k^*(\eta) u_k(\eta) \Big), \end{aligned} \tag{27}$$

where the last term corresponds to the vacuum average, which we extracted according to Eq. (25). It should be noted, that the summation over **k** in Eq. (27) is restricted by the value of the conformal Hubble constant. That is ad hoc definition because we discuss the creation of the particles under vacuum. The modes with the $|\mathbf{k}|<\mathcal{H}$ do not oscillate, and can not correspond to the real particles. Let us consider not only quantum average but also average in time for the concrete form of the functions $u_k(\eta)$ given by (24). That causes further simplification because it removes the oscillating terms. As a result, we come to

$$\bar{\rho}=\frac{1}{2a^4}\sum_{|\mathbf{k}|>\mathcal{H}}\frac{k^2}{\sqrt{k^2-\mathcal{H}^2}}\left(\alpha_k\alpha_k^*\Delta_k-\frac{1}{(\alpha_k-\alpha_k^*)^2\Delta_k}-1\right). \tag{28}$$

Let us minimize the energy density by choosing the corresponding Δ_k, which turns out to be equal $\Delta_k=|\alpha_k(\alpha_k^*-\alpha_k)|^{-1}$. This gives

$$\bar{\rho}=\frac{1}{2a^4}\sum_{|\mathbf{k}|>\mathcal{H}}\frac{k^2}{\sqrt{k^2-\mathcal{H}^2}}\left(2\alpha_k^*\alpha_k\sqrt{-(\alpha_k^*-\alpha_k)^2}-1\right). \tag{29}$$

Quantities α_k characterize phases of the functions $u_k(\eta)$ near singularity. It is convenient to represent them in the form $\alpha_k=r_k\exp\left(i(\pi/2+\theta_k)\right)$, where r_k and θ_k are reals.

As a result, we come to the final formula for the density of particles created from the wave packet set in singularity

$$\bar{\rho}=\frac{1}{2a^4}\sum_{|\mathbf{k}|>\mathcal{H}}\frac{k^2}{\sqrt{k^2-\mathcal{H}^2}}\left(\sec\theta_k-1\right)=\frac{1}{4\pi^2a^4}\int_{\mathcal{H}}^{\infty}\frac{k^4}{\sqrt{k^2-\mathcal{H}^2}}\left(\sec\theta_k-1\right)dk, \tag{30}$$

where integration has been changed by the summation $\sum_{\mathbf{k}}\to\int\frac{d^3\mathbf{k}}{(2\pi)^3}$.

Let us consider the model which allows obtaining arbitrary mean energy density taking

$$\sec\theta_k=1+\left(\mu k\right)^n, \tag{31}$$

where μ and n are some constants. As one may see from (31), the energy density is proportional to μ^n and, thus, can be set arbitrary. Besides, we shall consider that there exists momentums UV cut-off of the order of the Planck mass if some divergent integrals appear [10, 16]. It was shown that this cut-off is necessary to obtain a true value of the universe acceleration due to residual vacuum fluctuations [16].

3. The primordial spectrum of the energy density inhomogeneity. Let us calculate spectrum of the energy density

$$\hat{\rho}(\mathbf{r})=\frac{1}{2a^2}\left(\hat{\phi}'(\mathbf{r})^2+\left(\nabla\hat{\phi}(\mathbf{r})\right)^2\right). \tag{32}$$

As we consider the Gaussian wave packets C, the energy density in the different space points takes the form

$$<C|\hat{\rho}(\mathbf{r})\hat{\rho}(\mathbf{r}')|C>=\zeta(|\mathbf{r}-\mathbf{r}'|), \tag{33}$$

where

$$\zeta(r)=\sum_{\mathbf{k}}\sigma_k^2\exp\left(i\mathbf{kr}\right),\qquad \sigma_k^2=<C|\hat{\rho}_{\mathbf{k}}^+\hat{\rho}_{\mathbf{k}}|C>-<0|\hat{\rho}_{\mathbf{k}}^+\hat{\rho}_{\mathbf{k}}|0>. \tag{34}$$

Let us emphasize that no normal ordering is used in (34) because the summation is extended to all **k** domain and $\rho_{-\mathbf{k}}=\rho_{\mathbf{k}}^+$. In the equation (34) we again extract vacuum average ad hoc, because we are interested in the spectrum of the created particles under vacuum.

The Fourier components of energy density are expressed as

$$\hat{\rho}_{\mathbf{k}} = \frac{1}{V}\int_V \hat{\rho}(\mathbf{r})\exp(-i\mathbf{r}\mathbf{k})\, d^3\mathbf{r} = \frac{1}{2a^2}\sum_{\mathbf{q}} \hat{\varphi}_{\mathbf{q}}^{+'}\hat{\varphi}_{\mathbf{q}-\mathbf{k}}' + (\mathbf{q}-\mathbf{k})\mathbf{q}\hat{\varphi}_{\mathbf{q}}^{+}\hat{\varphi}_{\mathbf{q}-\mathbf{k}},$$
$$\hat{\rho}_{\mathbf{k}}^{+} = \frac{1}{2a^2}\sum_{\mathbf{q}} \hat{\varphi}_{\mathbf{q}-\mathbf{k}}^{+'}\hat{\varphi}_{\mathbf{q}}' + (\mathbf{q}-\mathbf{k})\mathbf{q}\hat{\varphi}_{\mathbf{q}-\mathbf{k}}^{+}\hat{\varphi}. \tag{35}$$

The steps of the previous section were: integration over $\mathcal{D}P_{\mathbf{q}}\mathcal{D}P_{\mathbf{q}}^{*}$, using Δ_k, which corresponds to the minimal energy density and, removing the oscillating terms. Here the calculations are the same, but more complicated, and can be done using Mathematica software. As a result, with the functions $u_k(\eta)$ given by (34), we come to

$$\sigma_k^2 = \langle C|\hat{\rho}_{\mathbf{k}}\hat{\rho}_{\mathbf{k}}^{+}|C\rangle - \langle 0|\hat{\rho}_{\mathbf{k}}\hat{\rho}_{\mathbf{k}}^{+}|0\rangle =$$
$$= \sum_{\substack{|\mathbf{q}|>\mathcal{H},\\ |\mathbf{q}-\mathbf{k}|>\mathcal{H}}} \frac{\left(\mathbf{q}(\mathbf{q}-\mathbf{k})\left(2\mathcal{H}^2 + \mathbf{q}(\mathbf{q}-\mathbf{k})\right) + q^2(\mathbf{q}-\mathbf{k})^2\right)(\sec\theta_q \sec\theta_{|\mathbf{q}-\mathbf{k}|} - 1)}{16a^8\sqrt{(q^2-\mathcal{H}^2)((\mathbf{q}-\mathbf{k})^2-\mathcal{H}^2)}}, \tag{36}$$

where summation on $\mathbf{q}$ is again restricted by considering only real particles. It is known that the inhomogeneity of the microwave background is relatively small. That restricts the value of the relative inhomogeneity given by the dimensionless quantity $k^3\sigma_k^2/\bar{\rho}^2$. Let us again to take the dependence (31). According to (30), $\bar{\rho}$ is proportional to the constant μ^n. At $n = 4$ the integral in Eq. (30) diverges logarithmically. In the general case, one has:

$$\bar{\rho} \sim \begin{cases} \frac{1}{a^4}\mathcal{H}^4\left(\frac{\mu}{\mathcal{H}}\right)^n\left(\frac{k_{\max}}{\mathcal{H}}\right)^{4-n}, & n<4, \\ \frac{1}{a^4}\mathcal{H}^4\left(\frac{\mu}{\mathcal{H}}\right)^4 \ln\left(\frac{k_{\max}}{\mathcal{H}}\right), & n=4, \\ \frac{1}{a^4}\mathcal{H}^4\left(\frac{\mu}{\mathcal{H}}\right)^n, & n>4, \end{cases} \tag{37}$$

where $k_{\max}$ is the UV cut-off of the order of the Planck mass. Analogous estimation for Eq. (36), when summation $\sum_{\mathbf{q}}$ is replaced by the integration $\int \frac{d^3\mathbf{q}}{(2\pi)^3}$, leads to

$$k^3\sigma_k^2|_{k\sim\mathcal{H}} \sim \begin{cases} \frac{\mathcal{H}^8}{a^8}\left(c_1\left(\frac{k_{\max}}{\mathcal{H}}\right)^{5-n}\left(\frac{\mu}{\mathcal{H}}\right)^n + c_2\left(\frac{\mu}{\mathcal{H}}\right)^{2n}\right), & 2<n<5, \\ \frac{\mathcal{H}^8}{a^8}\left(c_1\left(\frac{\mu}{\mathcal{H}}\right)^n + c_2\left(\frac{\mu}{\mathcal{H}}\right)^{2n}\right), & n>5. \end{cases} \tag{38}$$

Present day temperature of the universe microwave background is $T_0 = 2.73\,K = 2.35\cdot10^{-4}$ eV, the UV cut-off is the order of the Planck mass $k_{\max} \sim M_p = \sqrt{\frac{3}{4\pi G}} = 6\cdot10^{18}$ GeV, and the Hubble constant is $\mathcal{H} \sim 2.1\cdot10^{-33}$ eV. Energy density corresponding to the microwave background temperature is

$$\bar{\rho} \sim \frac{1}{a^4}T_0^4. \tag{39}$$

Using Eqs. (37), (39), we can rewrite Eq. (38) in terms of T_0

$$\frac{k^3\sigma_k^2}{\bar{\rho}^2}\Big|_{k\sim\mathcal{H}}\sim\begin{cases} c_1\dfrac{k_{\max}}{\mathcal{H}}\left(\dfrac{T_0}{\mathcal{H}}\right)^{-4}+c_2\left(\dfrac{k_{\max}}{\mathcal{H}}\right)^{2n-8}, \ 2<n<4, \\ c_1\dfrac{k_{\max}}{\mathcal{H}}\left(\ln\left(\dfrac{k_{\max}}{\mathcal{H}}\right)\right)^{-1}\left(\dfrac{T_0}{\mathcal{H}}\right)^{-4}+c_2\left(\ln\left(\dfrac{k_{\max}}{\mathcal{H}}\right)\right)^{-2}, \ n=4, \\ c_1\left(\dfrac{k_{\max}}{\mathcal{H}}\right)^{5-n}\left(\dfrac{T_0}{\mathcal{H}}\right)^{-4}+c_2, \ 4<n<5, \\ c_1\left(\dfrac{T_0}{\mathcal{H}}\right)^{-4}+c_2, \ n>5, \end{cases} \tag{40}$$

where c_1 and c_2 are some constants of the order of unity. The first observation is that the c_1-term is always suppressed by the multiplier $\left(T_0/\mathcal{H}\right)^{-4}$ and is negligible, despite the presence of the large multiplier $k_{\max}/\mathcal{H}$. The second observation is that the low relative inhomogeneity could be obtained if n is close to 4. In particular, one has $\dfrac{k^3\sigma_k^2}{\bar{\rho}^2}\sim\left(\ln\left(k_{max}/\mathcal{H}\right)\right)^{-2}\sim 2\cdot 10^{-4}$ for $n = 4$.

Let us calculate the form of spectrum. For low inhomogeneity, it is completely fixed by (31), because n should be very close to 4 and one has the integral

$$\sigma_k^2=\frac{\mu^{2n}}{16a^8}\int_{\substack{|\mathbf{q}|>\mathcal{H},\\ |\mathbf{q}-\mathbf{k}|>\mathcal{H}}}\frac{\left(\mathbf{q}(\mathbf{q}-\mathbf{k})\left(2\mathcal{H}^2+\mathbf{q}(\mathbf{q}-\mathbf{k})\right)+q^2(\mathbf{q}-\mathbf{k})^2\right)}{q^n(\mathbf{q}-\mathbf{k})^n\sqrt{(q^2-\mathcal{H}^2)((\mathbf{q}-\mathbf{k})^2-\mathcal{H}^2)}}\frac{d^3\mathbf{q}}{(2\pi)^3}, \tag{41}$$

which can be calculated using the method of Monte-Carlo. For $n\approx 4$ it is convergent at an upper limit and contains the only parameter $\mathcal{H}$. Spectrum $\mathcal{P}(k)=\dfrac{k^3\sigma_k^2}{\bar{\rho}^2}$ is shown in Fig. 1.

The spectrum is relatively flat but with the suppression at low comoving momentums k. It is interesting, that it is possible to obtain even more flat spectrum shown in Fig. 2 for $n = 4.11$, but the value of the relative inhomogeneity is large. For small inhomogeneity, the parameter n should be a bit less than four.

Conclusion. Studies of the Milne-like models seem attractive because they do not demand inflation. Moreover, all scales of interest always remain within the horizon during the evolution of universe. Let us remind that in the standard model of the radiation dominant universe with the inflation stage, a mode crosses horizon at the stage of inflation and then returns at the radiation or matter domination stage.

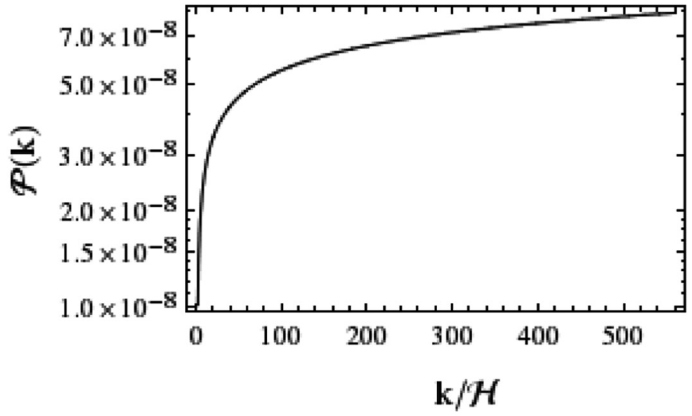

Fig. 1. The primordial spectrum of the relative density inhomogeneity $\mathcal{P}(k)=\dfrac{k^3\sigma_k^2}{\bar{\rho}^2}$ for parameter $n = 3.95$ in the formula (31)

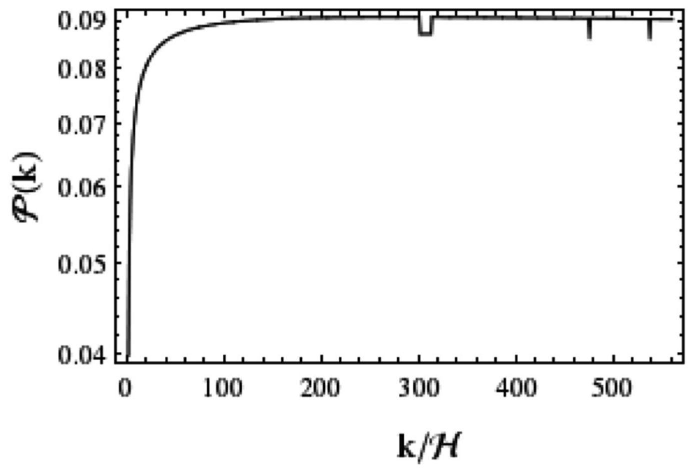

Fig. 2. The primordial spectrum of the relative density inhomogeneities $\mathcal{P}(k)=\frac{k^3\sigma_k^2}{\bar{\rho}^2}$ for parameter $n = 4.11$ in the formula (31)

It is shown that there exists the possibility to put any amount of matter into the cosmological singularity. If one demands that the relative value of inhomogeneity is small, the primordial spectrum has the fixed form, with the suppression of the large-scale modes. Because this spectrum corresponds to the scalar particles, the most evident candidate is the Higgs particles, which has been discovered recently. These particles decay into the photons and transfer the inhomogeneity to them. That is, the spectrum of scalar particles gives the initial conditions for the evolution of the photon modes. The initial conditions should be set at $\eta \sim 1/k$. The next step is to investigate the evolution of the photon spectrum and the massive matter spectrum in the Milne-like cosmology, but it is beyond the scope of the present lecture. Besides, there could be an analogous spectrum of the gravitational waves, which are equivalent to the massless scalar fields [16], but it does not undergo any change except for the cooling due to universe expansion.

The picture, presented in the lecture is very simplified because, for simplicity, we do not consider masses of the particles and spectrum oscillation omitting the oscillating terms in the formula (36).

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Information about the authors

Sergey L. Cherkas – Ph. D. (Physics and Mathematics), Senior Researcher, Institute for Nuclear Problems of the Belarusian State University (11, Bobruiskaya Str., 220050, Minsk, Republic of Belarus). E-mail: cherkas@inp.bsu.by

Vladimir L. Kalashnikov – Ph. D. (Physics and Mathematics), Senior Researcher, Institute of Photonics, Vienna University of Technology, (27/387, Gusshausstrasse, A-1040, Vienna, Austria). E-mail: vladimir.kalashnikov@tuwien.ac.at

Сведения об авторах

Черкас Сергей Леонидович – кандидат физико-математических наук, старший научный сотрудник, Институт ядерных проблем Белорусского государственного университета (ул. Бобруйская, 11, 220030, г. Минск, Республика Беларусь). E-mail: cherkas@inp.minsk.by

Калашников Владимир Леонидович – кандидат физико-математических наук, старший научный сотрудник, Венский технический университет (Gusshausstrasse 27/387, A-1040, г. Вена, Австрия). E-mail: vladimir.kalashnikov @tuwien.ac.at