ISSN 1561-2430 (Print)
ISSN 2524-2415 (Online)
UDC 530.12
https://doi.org/10.29235/1561-2430-2024-60-1-43-51

Received 05.03.2024
Поступила в редакцию 05.03.2024

Igor G. Dudko, Yuri P. Vyblyi

B. I. Stepanov Institute of Physics of the National Academy of Science of Belarus, Minsk, Republic of Belarus

FREUD – NAMBU COSMOLOGY WITH THE MASSLESS SCALAR FIELD

Abstract. Within the framework of the generalization of Freund – Nambu scalar-tensor theory of gravity, a massless scalar field is considered, the source of which is the trace of its own energy-momentum tensor. For the cosmological problem, numerical solutions of field equations were obtained, with the help of which the dependencies of the Hubble parameter and the photometric distance to the observed sources on red-shift were constructed. To the consistency of the models with observational data, contours of confidence intervals for model parameters were constructed.

Keywords: scalar field, scalar-tensor theory of gravity, the cosmological problem, Hubble parameter, red shift, confidence interval

For citation. Dudko I. G., Vyblyi Yu. P. Freund – Nambu cosmology with the massless scalar field. *Vestsi Natsyyanal'nai akademii navuk Belarusi. Seryya fizika-matematychnykh navuk = Proceedings of the National Academy of Sciences of Belarus. Physics and Mathematics series*, 2024, vol. 60, no. 1, pp. 43–51. https://doi.org/10.29235/1561-2430-2024-60-1-43-51

И. Г. Дудко, Ю. П. Выблый

Институт физики имени Б. И. Степанова Национальной академии наук Беларуси, Минск, Республика Беларусь

КОСМОЛОГИЯ ФРОЙНДА – НАМБУ С БЕЗМАССОВЫМ СКАЛЯРНЫМ ПОЛЕМ

Аннотация. В рамках обобщения скалярно-тензорной теории гравитации Фройнда – Намбу рассмотрено безмассовое скалярное поле, источником которого является след его собственного тензора энергии-импульса. Для космологической задачи получены численные решения полевых уравнений, с помощью которых построены зависимости параметра Хаббла и фотометрического расстояния до наблюдаемых источников от красного смещения. Для количественной оценки согласованности моделей с наблюдательными данными построены контуры доверительных интервалов изменения параметров моделей.

Ключевые слова: скалярное поле, скалярно-тензорная теория гравитации, космологическая задача, параметр Хаббла, красное смещение, доверительный интервал

Для цитирования. Дудко, И. Г. Космология Фройнда – Намбу с безмассовым скалярным полем / И. Г. Дудко, Ю. П. Выблый // Вес. Нац. акад. навук Беларусі. Сер. фіз.-мат. навук. – 2024. – Т. 60, № 1. – С. 43–51. https://doi.org/10.29235/1561-2430-2024-60-1-43-51

Introduction. Numerous current observations [1–3] lead to the confirmation that the Universe is expanding with the acceleration. As it is well known, the accelerated expansion is successfully described by the Λ*CDM*-model. However, the nature of Λ-term is unclear [4–7]. For this reason, along with other approaches scalar fields are often considered as an extension of general relativity to a scalar-tensor theory of gravitation. Also scalar fields are good candidates for the modeling of the dark energy (see, for example, [8, 9]).

At present, there are a lot of different classes of scalar-tensor models of a dark energy [10]. These models imply the equation of state parameter values $-1 < \omega_\varphi < -1/3$ for accelerated expansion of the Universe provided that the Universe contains only scalar field energy component. Besides, the dark energy the Universe contains the baryonic, cold dark matter and a negligible small portion of radiation which tend to slow down expansion of the Universe. This circumstance leaves only the narrow band of allowable values of the equation of state parameter ω that is close to minus one for the models above and also includes Λ*CDM*-model. However, this interval admits also values $\omega < -1$ which lead to some exotic scalar models with a negative kinetic term called phantom energy.

Nevertheless, there is a lot of freedom in choosing the scalar field Lagrangian. One of the possible approaches is a method for reconstructing the potential $V(\varphi)$ of a scalar field using behavior of function of density perturbations dependence on redshift in dust-like matter component provided that we know the Hubble constant value or using the luminosity distance as a function of redshift provided that we know the present density of dust-like matter estimated in the critical density units [11].

To get around the arbitrariness of choosing the Lagrangian of scalar field we use an assumption about the scalar field equation structure. As known, the Einstein's equations of the gravitational field can be obtained if one considers a linear massless tensor field in Minkowski space-time with the source being the stress-energy tensor of the field itself . We consider a scalar field with a similar property: we are assuming that the scalar field source is the trace of its own stress-energy tensor.

The task of obtaining such scalar field Lagrangian was considered many years ago by P. Freund and Y. Nambu in [12]. The boundary condition which they used was the requirement of the free scalar field Lagrangian when the scalar interaction constant vanishes. We will apply the same technique for the case of the massless scalar field but without using their assumption. In the frameworks of this approach we will derive the scalar field Lagrangian which contains two parameters: constant of the scalar interaction q and parameter C which relates to the minimum of scalar potential. Further, we will apply both models to the cosmology task and show that they satisfy the Hubble parameter dependence on red-shift and SNe Ia data with the appropriate choice of parameters.

Scalar field equation and the cosmological task. In this paper we will use the gravitational system of units $G = c = 1$. As it was mentioned, the Einstein's equations can be obtained as the equations of a linear massless tensor field with the source being the stress-energy tensor of the field itself in Minkowski space-time. We require that the equation of the scalar field interacting with the gravitational one has the same structure – the source of the scalar field is the trace of its own stress-energy tensor. For the massive scalar field we should obtain the equation of the following structure

$$(\Box - m^2)\varphi = qT^{\varphi}, \tag{1}$$

where the d'Alembertian[1] is $\Box \equiv -g^{\mu\nu}\nabla_\mu\nabla_\nu$, constants m and q mean the mass and the interaction constant of the scalar field. Note that hereinafter, all covariant quantities will refer to the Riemannian metric $g_{\mu\nu}$. The generalization of Freund and Nambu model was considered in [13]. The Lagrangian of this model for massless scalar field has the form

$$L = \frac{1}{2}\left(\frac{\partial_\mu\varphi\partial^\mu\varphi}{\Phi} + C\Phi^2\right), \tag{2}$$

where C is the constant which relates to the minimum of the scalar field potential, $\Phi = 1 + 2q\varphi$. Variation with respect to scalar field φ and to metric tensor $g_{\mu\nu}$ yields the equation and stress-energy tensor of the scalar field correspondingly

$$\Box\varphi = qT_\varphi = -q\left(\frac{\partial_\mu\varphi\partial^\mu\varphi}{\Phi} + 2C\Phi^2\right), \tag{3}$$

$$T^{\mu\nu}_\varphi = \frac{\partial^\mu\varphi\partial^\nu\varphi}{\Phi} - \frac{1}{2}g^{\mu\nu}\left(\frac{\partial_\alpha\varphi\partial^\alpha\varphi}{\Phi} + C\Phi^2\right). \tag{4}$$

Thus, the total Lagrangian of the model takes the form

$$L = L_g + L_\varphi + L_M, \tag{5}$$

where

$$L_g = R\sqrt{-g} \tag{6}$$

[1] The metric signature (+, –, –, –) was used.

is the Lagrangian of the gravitational field, L_φ is given by (2), $L_M(g_{\mu\nu}, Q_M)$ is the Lagrangian of the matter fields. Both Lagrangians L_φ have non-linear kinetic term which can be positive or negative depending on the scalar field value.

Now let us consider the application of both models to the late-time Universe cosmology. In this work we will consider the scalar field which does not interact with the matter. The system of equations with self-interacting massless scalar field (hereinafter – SIML model) takes the form

$$G^{\mu\nu} = 8\pi\left(\frac{\partial^\mu\varphi\partial^\nu\varphi}{\Phi} - \frac{1}{2}g^{\mu\nu}\left(\frac{\partial_\alpha\varphi\partial^\alpha\varphi}{\Phi} - C\Phi^2\right) + (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}\right), \quad (7)$$

$$\Box\varphi = -q\left(\frac{\partial_\mu\varphi\partial^\mu\varphi}{\Phi} + 2C\Phi^2\right), \quad (8)$$

$$\nabla_\mu T^{\mu\nu} = 0, \quad (9)$$

where $T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}$ is the stress-energy tensor of the perfect fluid which is the baryonic and cold dark matter. Here ε is the total energy density and p is the pressure which is equal to zero for the matter components. Further we will not take into account the radiation energy density since it is negligible at the present time and at those red-shifts that are necessary for the datasets under consideration.

For the spatially flat Friedman – Lemaitre – Robertson – Walker metric

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2) \quad (10)$$

the cosmological equations, which include Friedman equation, massless scalar field equation and covariant law of conservation for stress-energy tensor of matter, are

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3}\left(\frac{1}{2}\frac{\dot\varphi^2}{\Phi} - \frac{1}{2}C\Phi^2 + \varepsilon\right), \quad (11)$$

$$\ddot\varphi + 3H\dot\varphi - \frac{q\dot\varphi^2}{\Phi} - 2Cq\Phi^2 = 0, \quad (12)$$

$$\dot\varepsilon + 3H(\varepsilon + p) = 0. \quad (13)$$

Here $H \equiv \dot a / a$ is the Hubble parameter. Comparing the stress-energy tensor of the scalar field with the stress-energy tensor of the perfect fluid in the local-Lorentz coordinates, we can write the following expressions for the energy density and the scalar field pressure:

$$\varepsilon^\varphi = \frac{1}{2}\left(\frac{\dot\varphi^2}{\Phi} - C\Phi^2\right), \quad (14)$$

$$p^\varphi = \frac{1}{2}\left(\frac{\dot\varphi^2}{\Phi} + C\Phi^2\right). \quad (15)$$

If we identify density Ω_φ of the dark energy at the present stage with the energy density of the scalar field and specify the equation of state parameter w_φ, we can obtain the expressions for initial values $\varphi_0(H_0, \Omega_\varphi, \omega_\varphi, q)$ and $\dot\varphi_0(H_0, \Omega_\varphi, \omega_\varphi, q)$ as the functions of variable parameters

$$\varepsilon^\varphi(1 + w_\varphi) = \varepsilon_{cr}\Omega_\varphi(1 + \omega_\varphi) = \frac{3H_0^2\Omega_\varphi}{8\pi}(1 + \omega_\varphi) = \frac{\dot\varphi_0^2}{\Phi_0}, \quad (16)$$

$$\varepsilon^\varphi(1 - w_\varphi) = \varepsilon_{cr}\Omega_\varphi(1 - \omega_\varphi) = \frac{3H_0^2\Omega_\varphi}{8\pi}(1 - \omega_\varphi) = -C{\Phi_0}^2, \quad (17)$$

where we choose the present moment of time for the initial values of the scalar field and take this value as unity $t = 1$ due to homogeneity of time and $\Phi_0 = 1 + 2q\varphi_0$. Note that constant C will be excluded later, so we do not consider it as the variable parameter. From the system of algebraic equations (16)–(17) we obtain the expression for initial values of the scalar field

$$\varphi_0^{(1,2)} = -\frac{1}{2q} \pm \frac{1}{2q}\sqrt{-\frac{3H_0^2\Omega_\varphi}{8\pi C}(1-\omega_\varphi)}, \tag{18}$$

$$\dot{\varphi}_0^{(1,2)} = \pm\left(\pm\frac{3H_0^2\Omega_\varphi}{8\pi}(1+\omega_\varphi)\sqrt{-\frac{3H_0^2\Omega_\varphi}{8\pi C}(1-\omega_\varphi)}\right)^{1/2}, \tag{19}$$

where the signs before the square roots in (18) and signs inside the brackets in (19) are correlated. As it is seen from (18)–(19) and the structure of the kinetic term of the Lagrangian, two kinds of initial conditions correspond to different available regions of the scalar field dynamic. Indeed, let us take into account the definition of the equation of state parameter

$$\omega_\varphi = \frac{p}{\rho} = 1 - \frac{2V(\varphi)}{f(\varphi)\dot{\varphi}^2/2 + V(\varphi)}. \tag{20}$$

It is clear that the positive scalar factor $f(\varphi) > 0$ leads to restriction $w_\varphi > -1$ and the quintessence scalar field dynamics which corresponds to $\varphi > -1/2q$ and positive sign before the second term of the expression (18). The opposite sign, hence, corresponds to case $w_\varphi < -1$ and phantom dark energy dynamics. Also, it follows from (18) and (19) that we should use only negative values of constant C.

Since Λ-term in the Einstein's equations provides the exponential growth of the scalar factor and at the same time allows the observational data to be satisfied, we should simulate the same dynamics of the scalar factor by the scalar field. For this reason we will use the slow-roll approximation. This technique is commonly used in the inflationary scenario for the same purpose. In this case the energy density of a scalar field is the almost constant function of the scalar field variable at some interval of time which allows simulation of Λ-term in the right-hand side of the Friedman equation. This leads to the restrictions on the scalar field potential and its first derivative. Also we can neglect the second derivative of the scalar field variable $\ddot{\varphi}$ in the scalar field equation (12) at the current moment of time to establish the expression for parameter C as a function of the other parameters.

In the slow-roll mode the scalar field equation at the moment takes the form

$$3H_0\dot{\varphi}_0 = -V'(\varphi_0) = 2Cq{\Phi_0}^2. \tag{21}$$

Now we can substitute the initial values of formulas (18) and (19) in the previous equation and derive the expression for constant C as the function of parameters H_0, Ω_φ, ω and q

$$C = -\frac{27H_0^2\pi(1+\omega)^2}{2\Omega_\varphi q^4(1-\omega)^3}. \tag{22}$$

Let us note that the application of the slow-roll approximation made it possible to reduce the number of free parameters of the model. Also it helps us to determine the exact sign of the initial condition (19) using the equation (21) which was written for the current moment of time.

Thus, for SIML model we have four free parameters H_0, Ω_φ, ω and q. The initial conditions for the cosmological equation system (11)–(13) have the form (18), (19) for the scalar field and its first derivative and

$$a_0 = 1, \quad \dot{a}_0 = H_0, \quad \varepsilon_0 = \frac{3H_0^2(1-\Omega_\varphi)}{8\pi}$$

for the other functions. Here we took advantage of the fact that we can choose any predetermined value for the scale factor in the spatially flat Universe as well as the fact that the sum of the energy density parameters is equal to unity $\sum_i \Omega_i = 1.$ Note also that we will consider the initial conditions only for the quintessence scalar field model.

Observational data and parameter estimation. To obtain best-fit values and confidence intervals for considered parameters we need to get numerical solutions for scale factor $a(t)$ from the system of cosmological equations. Then we obtain the Hubble parameter and distance modulus dependence on the redshift and compare these theoretical values for each fixed set of parameters with the selected sets of observational data.

Measurements of the Hubble parameter $H(z)$ for different redshifts z with $N_H = 57$ data points are presented in compact form in [14]. These points were obtained by two methods: the line-of-sight BAO method and the method based on the cosmic chronometers, this is the estimations from differential ages Δt of galaxies using the following relation

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt} \simeq -\frac{1}{1+z}\frac{\Delta z}{\Delta t}. \tag{23}$$

For both models, we calculate function χ_H^2, assuming that the errors in the data have a Gaussian distribution

$$\chi_H^2(p) = \sum_{i=1}^{N_H}\left(\frac{H_{th}(z_i, p) - H_{obs}(z_i)}{\sigma_i}\right)^2, \tag{24}$$

where $p \equiv (p_1, p_2, ...p_k)$ is the set of the model parameters.

Further we use the Pantheon dataset compilation of SNe Ia which contains 1048 data points of the redshift and corresponding luminosity distance modulus and error bars in the interval $0.01 < z < 2.3$. To obtain a theoretical value of distance modulus μ_{th} at the required redshift we should calculate the luminosity distance which in the spatially flat Universe has the form

$$D_L(z, p) = \frac{c(1+z)}{H_0}\int_0^z \frac{dz'}{H(z', p)} \tag{25}$$

for each fixed set of model parameters. Using the required values of the red shift z_i from datasets we can get the theoretical value of distance modulus μ_{th}

$$\mu_{th}(z, p) = 5\lg D_L(z, p) + 25 = 5\lg d_L(z, p) + \mu_0, \tag{26}$$

where $\mu_0 = 42.38 - 5\lg h,$ $d_L \equiv H_0 D_L / c$ and $h \equiv H_0 / 100.$ Further, again assuming that the errors in the data have Gaussian distribution, it wold be possible to construct the standard χ_{SN}^2 function, however there will be nuisance parameter μ_0 or, which is the same, h. We minimize the contribution of this parameter, following [14]. In the explicit form χ_{SN}^2 function will take the form

$$\chi_{SN}^2(p) = \sum_{i=1}^{N}\frac{\left(\mu_{obs}(z_i) - 5\lg d_L(z_i, p) - \mu_0\right)^2}{\sigma_i^2}, \tag{27}$$

where nuisance parameter μ_0 does not depend on the data points in the set. After opening the brackets we obtain the next expression $\chi_{SN}^2(p) = A - 2\mu_0 B + \mu^2 C,$ where

$$A(p) \equiv \sum_{i=1}^{N}\frac{\left(\mu_{obs}(z_i) - 5\lg d_L(z_i, p)\right)^2}{\sigma_i^2}, \tag{28}$$

$$B(p) \equiv \sum_{i=1}^{N}\frac{\left(\mu_{obs}(z_i) - 5\lg d_L(z_i, p)\right)}{\sigma_i^2}, \tag{29}$$

$$C \equiv \sum_{i=1}^{N} \frac{1}{\sigma_i^2}, \tag{30}$$

which has the minimum at the point $\mu_0 = B/C$. Substituting the resulting value in the original equality we get a new estimator

$$\tilde{\chi}^2_{SN}(p) = A(p) - \frac{B^2(p)}{C}. \tag{31}$$

Note that with such approach, the minimums of both functions will coincide $\chi^2_{min} = \tilde{\chi}^2_{min}$.

Calculations and results. The set of our cosmological parameters is (H, Ω_M, ω, q). For the numerical solution of the cosmological equations it is necessary to make it dimensionless. This procedure was performed with the help of the Hubble constant numerically equal to $H_0 = 100\ s^{-1}$.

Free model parameters the vary in the following ranges

$$h \in [0.5; 0.9], \tag{32}$$

$$\Omega_M \in [0.1; 0.7], \tag{33}$$

$$\omega \in [-0.999; -0.35], \tag{34}$$

$$q \in [0.01; 1]. \tag{35}$$

The interval for the Hubble parameter (32) is taken that one essentially to cover values ranges from $67.37 \pm 0.54\ \text{km}/(\text{s}\cdot\text{Mpc})$ to $H = 73.3^{+1.7}_{-1.8}\ \text{km}/(\text{s}\cdot\text{Mpc})$ and $74.03 \pm 1.42\ \text{km}/(\text{s}\cdot\text{Mpc})$, which were obtained by Planck, *H0LiCOW* and *SH0ES* collaborations. Note that this divergence in values of the Hubble constant, obtained by different methods is the so-called H_0-tension problem. In the interval (33) for Ω_M any values close to zero are excluded, so as not to consider the Universe practically without a matter. In the interval (34) for equation state parameter the right boundary is chosen so that it satisfies the condition $\omega < -1/3$. In (35) we assumed that the value of constant q should not exceed the value of gravitational constant $G = 1$. However, as it will be shown below, the function χ^2 practically don't depend on the parameter q. The last circumstance is connected, apparently, with the fact that we doesn't consider in this work the interact between a scalar field and a matter.

Further, for the sets of parameters from intervals (32)–(35) the systems of cosmological equations were solved. We used the initial conditions (18), (19) and energy density of matter. From the obtained solutions dependence of the Hubble parameter on the red shift $H_{th}(z,p)$ was constructed. Further, with the help of this dependence and the theoretical functions for distance modulus $\mu_{th}(z,p)$ were constructed. Using observational data for $H_{obs}(z_i)$ from [14], Pantheon dataset [15] of $\mu_{obs}(z_i)$ and corresponding theoretical values $H_{th}(z,p)$, $\mu_{th}(z,p)$, the total function $\tilde{\chi}^2_{H+SN}$ was obtained.

For MLSI model the results of calculations are shown in Fig. 1. All graphs were obtained by the method of minimization of function $\chi^2_{H+SN} \equiv \chi^2_\Sigma$. Fig. 1 contains 1σ, 2σ and 3σ confidence regions on the parameters planes and one-dimensional parameters distributions of χ^2_Σ. As it was noted above, the confidence regions in plains $H - \omega$, $H - \Omega$ and $\Omega - \omega$ correspond to areas $\chi^2_{\min} + 2.30$, $\chi^2_{\min} + 6.17$ and $\chi^2_{\min} + 11.8$, where $\chi^2_{\min}$ is the optimal value of functions $\chi^2(p_1, p_2)$, which are obtained with the help of minimization of function χ^2_Σ on remaining parameters – $\chi^2(H,\Omega) = \min_{\omega,q}\chi^2_\Sigma$, $\chi^2(H,\omega) = \min_{\Omega,q}\chi^2_\Sigma$ and $\chi^2(\Omega,\omega) = \min_{H,q}\chi^2_\Sigma$ accordingly. One-dimensional parameters distributions of $\chi^2(p_1)$ are obtained in a similar way – the minimization by three parameters, for example $\chi^2(H) = \min_{\Omega,\omega,q}\chi^2_\Sigma$.

The best-fit value of the Hubble parameter with 1σ confidence interval is $H = 68.88 \pm 0.69\ \text{km}/(\text{s}\cdot\text{Mpc})$. 1σ error bar is completely within the range of $H = 69.80 \pm 1.90\ \text{km}/(\text{s}\cdot\text{Mpc})$, which was obtained by CCHP. But the error bars of the results from Planck, *H0LiCOW* and *SH0ES* collaborations do not include the error bar of the Hubble parameter which yields the MLSI model. Value $H = 68.88 \pm 0.69\ \text{km}/(\text{s}\cdot\text{Mpc})$ is in 1.7σ tension with Planck H_0 value. The differences between *H0LiCOW* and *SH0ES* are in 2.3σ and 3.3σ tension correspondingly. Note that the best-fit Hubble parameter value as a whole is closer to

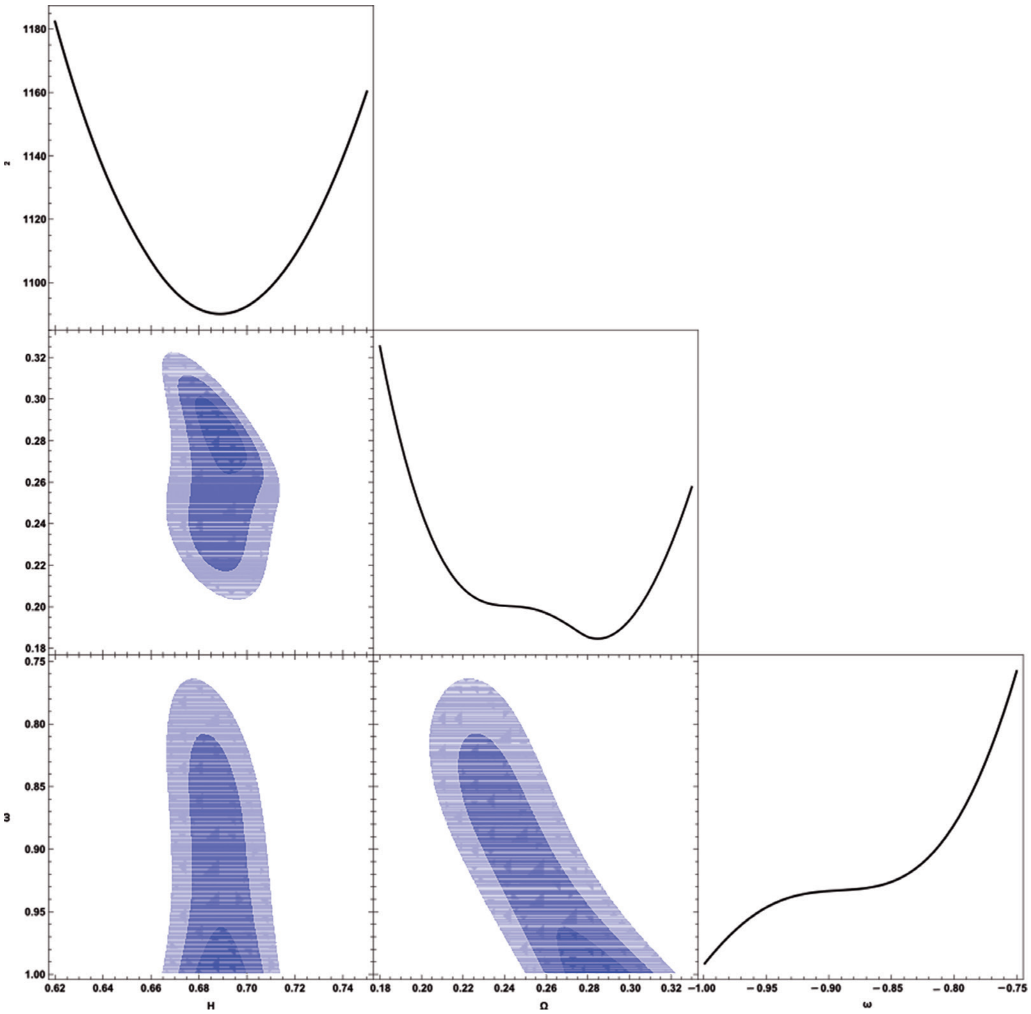

Fig. 1. The graphs show 1σ, 2σ and 3σ confidence regions on the parameters planes and one-parameter distributions which were obtained from the cumulative distribution χ^2_{H+SN}

the result Planck collaboration and in the interval 2σ captures it too. The optimal value and 1σ error bar of the matter energy density and equation of the state parameter of scalar field are $\Omega_m = 0.285 \pm 0.011$ and $\omega = -0.999^{+0.028}$. Parameter Ω_m has 2.3σ discrepancy compared to its Planck value $\Omega_m = 0.315 \pm 0.007$. Here and henceforth we will not make any comparison with parameter Ω_m, obtained by *H0LiCOW* collaboration for Λ*CDM*-model because of its error bar for time-delay cosmography only is an order of magnitude more than we obtained for our models and always includes them. As it was mentioned above, the equation of state parameter cannot equal to minus one, however as the best-fit value shows, it tends to this value in the slow-role approximation. Also parameter ω has no lower bound 1σ, because it is at the left border of the range of values that is allowed for models with a positive kinetic term in the Lagrangian. The optimal value of these parameters is achieved with the value of function $\chi^2_\Sigma = 1090.05$ or, in terms of reduced chi square, $\chi^2_{\text{reduced}} = 0.99$. From graphs in Fig. 1 it's clear that confidence regions are stretched in the direction of increasing values of the parameter of the equation of state and in the direction of decreasing values of matter density parameter. This is due to the influence of competition between two minima – global and local – for function χ^2_H to the procedure of minimizing of total function χ^2_{H+SN}. Global minimum χ^2_H is achieved at parameter values, $H = 63.41$, $\Omega_M = 0.288$ and $\omega = -0.615$ while the local one at $H = 70.02$, $\Omega_M = 0.264$ and $\omega = -0.999$. The addition of χ^2_{SN} function partially eliminates this problem and function χ^2_{H+SN} already has one global minimum. However it is not possible to completely avoid the influence of competition, which is reflected both on two-dimensional distributions in the form of extrusion of "tails", and on one-dimensional distributions of parameters Ω and ω.

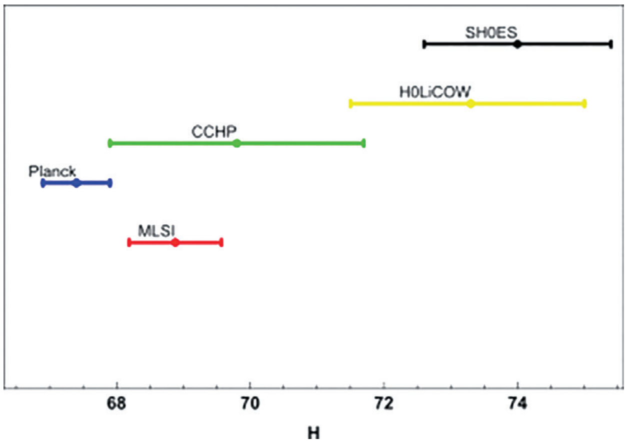

Fig. 2. The graph shows the values of the Hubble parameter with error bars for the considered model and those measured by different collaborations

Conclusion. In this article, we considered the scalar-tensor theory of gravity with self-acting scalar field. By analogy with Freund and Nambu model, who considered the massive scalar field with the source in the form of own energy-momentum trace, we have obtained the Lagrangian and the equation of such massless scalar field with the additional term. Further, in application to the cosmological problem, the initial values for scalar field, its first derivative, scale factor and matter energy density were defined. Due to the alternating sign of the kinetic terms the scalar field can model both the quintessence dynamics with the parameters of the equation of state $\omega_{DE} > -1$, and phantom dynamics,where the parameter of the equation state takes values $\omega < -1$. It is shown that the initial values of the scalar field are divided into two classes – values corresponding to the phantom area $\varphi < -1/2q$ and the values of $\varphi > -1/2q$ corresponding to the quintessence. The use of the slow roll approximation for the initial time moment for a scalar field allows expressing one parameter in terms of the others.

Further, using the data from cosmic chronometers measurements of the Hubble parameter the function $\chi^2_{H+SN}(p)$ was built. Using the procedure of minimization by two and three parameter two- and one-parameter distributions of $\chi^2(p_i, p_j)$ and $\chi^2(p_i)$ were obtained. Using these distributions, we have constructed the confidence regions for the parameters pair under consideration and obtained the 1σ error bar for each parameter separately. Summarizing, one can say that considered model is in good agreement with the observable data for the Hubble constant. The optimal values and error bars of all model lie within the 1σ confidence of CCHP result and have 1.7σ difference with respect to Planck H_0 value. In the context of H_0-tension problem it is worth noting that the best-fit values of H_0 parameter lies in the region of lower observational values of the Hubble parameter.

Acknowledgements. We would like to thank Prof. G. S. Sharov for useful discussions and clarifications of some computational points during this work. The work was supported by grants no. Ф22МЦ-005 and no. Ф22М-052 of the Belarusian Republican Foundation for Fundamental Research.

Благодарности. Авторы выражают благодарность профессору Г. С. Шарову за полезные обсуждения и разъяснения в ходе работы некоторых вычислительных вопросов. Работа выполнена при поддержке Белорусского республиканского фонда фундаментальных исследований (гранты № Ф22МЦ-005, Ф22М-052).

References

1. Riess A. G., Filippenko A. V., Challis P., Clocchiatti A., Diercks A., Garnavich P. M. [et al.] (Supernova Search Team). Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. *The Astronomical Journal*, 1998, vol. 116, pp. 1009–1038. https://doi.org/10.1086/300499

2. Perlmutter S., Aldering G., Goldhaber G., Knop R. A., Nugent P., Castro P. G. [et al.] (Supernova Cosmology Project). Measurements of Ω and Λ from 42 High-Redshift. Supernovae. *The Astronomical Journal*, 1998, vol. 517, pp. 565–586. https://doi.org/10.1086/307221

3. Eisenstein D. J., Zehavi I., Hogg D. W., Scoccimarro R., Blanton M. R., Nichol R. C., Scranton R. [et al.] (SDSS). Detection of the Baryon Acoustic Peak in the Large-Scale Correlation Function of SDSS Luminous Red Galaxies. *The Astronomical Journal*, 2005, vol. 633, pp. 560–574. https://doi.org/10.1086/466512

4. Sahni V., Starobinski A. The Case for a Positive Cosmological Lambda Term. *International Journal of Modern Physics D*, 2000, vol. 9, no. 4, pp. 373–444. https://doi.org/10.1142/S0218271800000542

5. Padmanabhan T. Cosmological constant – the weight of the vacuum. *Physics Reports*, 2003, vol. 380, no. 5–6, pp. 235–320. https://doi.org/10.1016/S0370-1573(03)00120-0

6. Matos T., Guzman F. S. On the spacetime of a galaxy. *Classical and Quantum Gravity,* 2001, vol. 18, no. 23, pp. 5055–5064. https://doi.org/10.1088/0264-9381/18/23/303

7. Peebles P. J. E., Ratra B. The cosmological constant and dark energy. *Reviews of Modern Physics*, 2003, vol. 75, pp. 559–606. https://doi.org/10.1103/revmodphys.75.559

8. Copeland E. J., Sami M., Tsujikawa S. Dynamics of dark energy. *International Journal of Modern Physics D*, 2006, vol. 15, no. 11, pp. 1753–1936. https://doi.org/10.1142/s021827180600942x

9. Belinchón J. A., Harko T., Mak M. K. Exact Scalar-Tensor Cosmological Solutions via Noether Symmetry. *Astrophysics and Space Science*, 2016, vol. 361, no. 2, p. 52. https://doi.org/10.1007/s10509-015-2642-7

10. Scherrer R. J., Sen A. A. Thawing quintessence with a nearly flat potential. *Physical Review D*, 2008, vol. 77, 083515. https://doi.org/10.1103/PhysRevD.77.083515

11. Starobinsky A. A. How to determine an effective potential for a variable cosmological term. *Journal of Experimental and Theoretical Physics Letters*, 1998, vol. 68, pp. 757–763. https://doi.org/10.1134/1.567941

12. Freund P., Nambu Y. Scalar Fields Coupled to the Trace of the Energy-Momentum Tensor. *Physical Review*, 1968, vol. 174, pp. 1741–1747. https://doi.org/10.1103/physrev.174.1741

13. Dudko I. G., Vyblyi Yu. P. Scalar field with the source in the form of the stress-energy tensor trace as a dark energy model. *Gravitation and Cosmology*, 2016, vol. 22, pp. 368–373. https://doi.org/10.1134/s020228931604006x

14. Sharov G. S., Vasilev V. O. How predictions of cosmological models depend on Hubble parameter data sets. *Mathematical Modelling and Geometry*, 2018, vol. 6, no, 1, pp. 1–20. https://doi.org/10.26456/mmg/2018-611

15. Scolnic D. M., Jones D. O., Rest A., Pan Y. C., Chornock R., Foley R. J., Huber M. E. [et al.] The Complete Light-curve Sample of Spectroscopically Confirmed SNe la from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample. *The Astronomical Journal*, 2018, vol. 859, no. 2, p. 101. https://doi.org/10.3847/1538-4357/aab9bb

Information about the authors

Igor G. Dudko – Researcher, B. I. Stepanov Institute of Physics of the National Academy of Sciences of Belarus (68-2, Nezavisimosti Ave., 220072, Minsk, Republicof Belarus). E-mail: igordudko89@gmail.com

Yuri P. Vyblyi – Ph. D. (Physics and Mathematics), Leading Researcher, B. I. Stepanov Institute of Physics of the National Academy of Sciences of Belarus (68-2, Nezavisimosti Ave., 220072, Minsk, Republic of Belarus). E-mail: vyblyi@gmail.com

Информация об авторах

Дудко Игорь Георгиевич – научный сотрудник, Институт физики имени Б. И. Степанова Национальной академии наук Беларуси (пр. Независимости, 68-а, 220072, Минск, Республика Беларусь). E-mail: igordudko89@gmail.com

Выблый Юрий Петрович – кандидат физико-математических наук, ведущий научный сотрудник, Институт физики имени Б. И. Степанова Национальной академии наук Беларуси (пр. Независимости, 68-2, 220072, Минск, Республика Беларусь). E-mail: vyblyi@gmail.com