ISSN 1561-2430 (Print)
ISSN 2524-2415 (Online)

ФИЗИКА
PHYSICS

UDC 530.1 Received 16.01.2024
https://doi.org/10.29235/1561-2430-2024-60-3-225-232 Поступила в редакцию 16.01.2024

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PRIMORDIAL BLACK HOLES IN THE EARLY UNIVESE, QUANTUM-GRAVITATIONAL CORRECTIONS AND INFLATIONARY COSMOLOGY

Abstract. In essence, primordial black holes generated in the early Universe as a result of a gravitational collapse of the high-density matter are detectors of the processes proceeding in it. As these black holes are generated at high energies (close to the Planck energies) and their radii are small, there is a need to take into consideration the quantum-gravitational corrections for them. In this paper, within the scope of the Generalized Uncertainty Principle, the author continues a study of the quantum-gravitational corrections and their contributions into the inflationary parameters for primordial black holes in the pre-inflationary epoch. Specifically, within this pattern, the author considers a case of Hawking's radiation (evaporation) for the above-mentioned black holes and derives formulae for the corresponding changes ("shifts") of the basic inflation parameters. In all cases the expressions for the corresponding correction of *e*-foldings in an inflation model have been found. In conclusion the main problems for further studies are formulated.

Keywords: primordial black holes, inflationary cosmology, quantum-gravitational corrections

For citation. Shalyt-Margolin A. E. Primordial black holes in the early Univese, quantum-gravitational corrections and inflationary cosmology. *Vestsi Natsyyanal'nai akademii navuk Belarusi. Seryya fizika-matematychnykh navuk = Proceedings of the National Academy of Sciences of Belarus. Physics and Mathematics series*, 2024, vol. 60, no. 3, pp. 225–232. https://doi.org/10.29235/1561-2430-2024-60-3-225-232

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ПЕРВИЧНЫЕ ЧЕРНЫЕ ДЫРЫ В РАННЕЙ ВСЕЛЕННОЙ, КВАНТОВО-ГРАВИТАЦИОННЫЕ ПОПРАВКИ И ИНФЛЯЦИОННАЯ КОСМОЛОГИЯ

Аннотация. Первичные черные дыры, возникающие в ранней Вселенной вследствие гравитационного коллапса материи высокой плотности, являются по сути детекторами происходящих в ней процессов. Так как эти черные дыры рождаются в высоких энергиях (близких к планковским) и радиусы их малы, то для них необходим учет квантово-гравитационных поправок. В настоящей работе в рамках обобщенного принципа неопределенности продолжено начатое автором ранее исследование квантово-гравитационных поправок и их вкладов в значения инфляционных параметров для первичных черных дыр в доинфляционную эпоху. В частности, в этой картине рассмотрен случай излучения (испарения) Хокинга для вышеуказанных черных дыр и получены явные формулы для соответствующих «сдвигов» основных параметров инфляции. Во всех случаях найдены выражения для соответствующей коррекции *e*-фолдингов в инфляционной модели. Сформулированы основные проблемы для дальнейшего исследования.

Ключевые слова: первичные черные дыры, инфляционная космология, квантово-гравитационные поправки

Для цитирования. Шалыт-Марголин, А. Э. Первичные черные дыры в ранней Вселенной, квантово-гравитационные поправки и инфляционная космология / А. Э. Шалыт-Марголин // Вес. Нац. акад. навук Беларусі. Сер. фіз.-мат. навук. – 2024. – Т. 60, № 3. – С. 225–232. https://doi.org/10.29235/1561-2430-2024-60-3-225-232

Introduction. As it is shown in [1–3], the so-called primordial black holes (pbh) may be formed in the early Universe. The most common mechanism for the formation of pbh is a gravitational collapse of the high-density matter caused by the cosmological perturbations arising (e. g. in the process of inflation but not necessarily) in the early Universe [4]. Studying of pbhs is very important because their parameters in essence are ”detectors” of the processes proceeding in the early Universe. Of interest are some results obtained during the relevant studies, for example, a sufficiently accurate estimate of the mass pbh $M(t)$ formed in the period of time t since the Big Bang [5–7]

$$M_H(t) \approx \frac{c^3 t}{G} \approx 10^{15}\left(\frac{t}{10^{-23}\ \text{s}}\right)\text{g}. \tag{1}$$

This estimate points to the fact that pbh has a wide range of masses, beginning from the Planck mass $M(t_M) \approx 10^{-5}\,\text{g}$ for the Planckian time $t_M = t_p \approx 10^{-43}\,\text{s}$. The quantum-gravitational corrections (qgcs) of such pbhs are significant.

A wide list of the literature devoted to studies of pbhs is reviewed in [4]. A great interest to pbhs is also associated with the fact that, according to the recent data, these objects are considered as one of the constituent elements of the dark matter. However, the works devoted to consideration of qgcs are few. In particular, the work [8], within the scope of an inflationary model, presents a study of the scalar cosmological perturbations associated with small-radius pbhs arising before the beginning of inflation. It is clear that for such pbhs, due to formula (1), taking into consideration of qgcs is important but this is not done in [8].

Note that by the present time there is no self-consistent theory of quantum gravity. Still, there are several approaches to such a theory, the principal results of which are in agreement and the correctness of which is beyond question [9]. One of these approaches is associated with replacement of the Heisenberg Uncertainty Principle (HUP) by the Generalized Uncertainty Principle (GUP) on going to high (Planck’s) energies.

The present paper is a continuation of a study presented in [10], where the author begins to study consideration of qgcs for primordial black holes with small radii of horizon in the pre-inflationary epoch. As compared to [10], qgcs are taken into consideration for small-radius pbhs in the process of Hawking’s radiation in inflationary models. Using such a pattern, within the scope of GUP, explicit formulae are derived for (quantum) shifts of the basic inflationary parameters and in all cases the expressions for the corresponding correction of e-foldings in an inflation model have been found. In what follows the normalization $c = \hbar = 1$ is used, for which we have $G = l_p^2$.

Primordial black holes in the early Universe, inflation parameters and initial results. The metric of a Schwarzschild black hole is of the form (formula (7.31) in [11])

$$ds^2 = -\left(1 - \frac{2MG}{r}\right)dt^2 + \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 + r^2 d\Omega^2, \tag{2}$$

where M is the mass of this black hole.

During studies of the early Universe for pbh the Schwarzschild metric (2) is replaced by the Schwarzschild-de Sitter (SdS) metric [8], allotted in our consideration to black holes in the pre-inflationary epoch

$$ds^2 = -f(\tilde{r})dt^2 + \frac{d\tilde{r}^2}{f(\tilde{r})} + \tilde{r}^2 d\Omega^2, \tag{3}$$

where $f(\tilde{r}) = 1 - 2GM/\tilde{r} - \Lambda\tilde{r}^2/3 = 1 - 2GM/\tilde{r} - \tilde{r}^2/L^2$, $L = \sqrt{3/\Lambda}$, M is the mass of a black hole, $\tilde{r}$ is the small quantity, and Λ is the cosmological constant.

In general, such a black hole may have two horizons corresponding to two different zeros $f(\tilde{r})$: event horizon of a black hole and a cosmological horizon. This is in particular the case when the quantity M is small [12, 13]. Just this case is of interest for us because we study small-radius black holes in the early Universe (1). Then event horizon radius of a black hole having the metric (3) takes the following form (formula (9) in [14]):

$$r_H = 2GM\left[1 + \left(\frac{r_M}{L}\right)^2\right], \tag{4}$$

where $r_M = 2GM$ is the event horizon radius of a Schwarzschild black hole (2) and, as $L \gg r_M$, the condition $r_H = r_M$ is fulfilled to a high accuracy. It should be noted that the event horizon radius r_M of a Schwarzschild black hole (2) takes the form $r_M = 2GM/c^2$ [11] but, due to the normalization adopted in the paper, this quantity is equal to $2GM$.

In the general case in cosmology, in particular inflationary one, the metric (3) is given in terms of the conformal time η [8]:

$$ds^2 = a^2(\eta)\left\{-d\eta^2 + \left(1+\frac{\mu^3\eta^3}{r^3}\right)^{4/3}\left[\left(\frac{1-\mu^3\eta^3/r^3}{1+\mu^3\eta^3/r^3}\right)^2 dr^2 + r^2 d\Omega^2\right]\right\}, \quad (5)$$

where $\mu = (GMH_0/2)^{1/3}$ and usually it is assumed as in [8] that $\mu = \text{const}$ and H_0 is the de Sitter – Hubble parameter, whereas the scale factor a is a function of the conformal time η (see formula (2.9) in [15] and p. 4 in [8]):

$$a(\eta) = -1/(H_0\eta), \qquad \eta < 0. \quad (6)$$

Here $\tilde{r} = r$ satisfies the condition $r_0 < r < \infty$ and the value of $r_0 = \mu\eta$ in the frame of reference (5) conforms to singularity of the black hole [8].

Due to the above-mentioned formulae, μ may be given as

$$\mu = (r_M H_0/4)^{1/3}, \quad (7)$$

where $\tilde{r} = r_M$ is the radius of a black hole with the Schwarzschild-de Sitter(SdS)-metrics (3),which (see the beginning of this section) to a high accuracy is the radius of the canonical Schwarzschild black hole (2).

To calculate qgcs for black holes with (SdS)-metrics (3) in [10], we have used the approach associated with the introduction of the Generalized Uncertainty Principle (GUP) at Planck's scales [16]

$$(\delta x)(\delta p) \geq \frac{\hbar}{2} < \exp\left(\frac{\alpha^2 l_p^2}{\hbar^2} p^2\right) > \quad (8)$$

which, on keeping of the leading term, gives the first-order GUP [16]:

$$(\delta x)(\delta p) \geq \frac{\hbar}{2}\left(1 + \frac{\alpha^2 l_p^2}{\hbar^2}(\delta p)^2\right). \quad (9)$$

Then we have the Planckian black hole (further referred to as "minimal"), which has the minimal mass M_0 and the minimal event-horizon radius $r_{\min}$ that is the theoretical minimal length $l_{\min}$ (formula (20) in [16]):

$$r_{\min} = (\delta x)_0 = \sqrt{\frac{e}{2}}\alpha l_p, \quad M_0 = \frac{\alpha\sqrt{e}}{2\sqrt{2}} m_P, \quad (10)$$

where α is the model-dependent parameter on the order of 1, e is the base of natural logarithms, and $r_{\min} \propto l_p$, $M_0 \propto m_P$.

First GUP (9) has been involved in a superstring theory [17], later it was derived in other approaches used to study a quantum theory at Planck's energies [18–22]. As compared to HUP, the high-energy term in the right-hand side of formula (9) is associated with allowance for the gravitational interactions which should not be ignored at Planck's scales. In the limit of low energies, $E \ll E_p$, this term becomes vanishingly small. Despite the fact that commutation relations at the energies $E \ll E_p$ are in general dependent on the metric (for example, formula (1) in [18]), GUP in fomula (9) arises at Planck's scales irrespective of the space-time geometry at low energies [18, 19, 22, 23].

Within the scope of GUP (8), going from the black-hole temperature T calculated in a semiclassical approximation to the same quantity T_q calculated with due regard for qgcs will be given as follows [16]:

$$\left(T = \frac{1}{8\pi G}\right) \to T_q = \frac{1}{8\pi G}\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) = \frac{1}{8\pi G}\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{r_{\min}}{r_M}\right)^2\right)\right). \quad (11)$$

Here $W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)$ value at the corresponding point of the Lambert W-function $W(u)$ satisfying the equation (formulae (1.5) in [24] and (9) in [16])

$$W(u)e^{W(u)} = u. \tag{12}$$

$W(u)$ is the multifunction for complex variable $u = x + iy$. However for real $u = x, -\frac{1}{e} \le u \le 0$. $W(u)$ (in present picture) is the single-valued continuous function, having two branches denoted by $W_0(u)$ and $W_{-1}(u)$ [24].

It is clear, that for a great black hole having large mass M and great event horizon area M, the deformation parameter $\frac{1}{e}\left(\frac{M_0}{M}\right)^2$ is vanishingly small and close to zero. Then a value of $W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)$ is also close to $W(0)$. As it is seen, $W(0) = 0$ is an obvious solution for the equation (12). We have

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) \approx 1. \tag{13}$$

So, a black hole with great mass $M \gg m_P$ necessitates no consideration of qgcs.

But in the case of small black holes we have

$$\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right) > 1. \tag{14}$$

R e m a r k 1. *The quantum-gravitational correction qgcs* (11) *presents a deformation (or more exactly, the quantum deformation) of a classical black-holes theory from the viewpoint of the paper* [25] *with the deformation parameter A_0/A, where $A_0 = 4\pi r_{\min}^2$ is the area of the event-horizon surface for a "minimal" black hole, $A_0 = 4\pi r_M^2$ is the same area for the black hole under study.*

Then we have

$$\frac{A_0}{A} = \frac{4\pi r_{\min}^2}{4\pi r_M^2} = \frac{l_{\min}^2}{r_M^2}. \tag{15}$$

It should be noted that this deformation parameter

$$\frac{l_{\min}^2}{r_M^2} \doteq \alpha_{r_M} \tag{16}$$

has been introduced by the author in his earlier works [26–29], *where he studied deformation of quantum mechanics at Planck's scales in terms of the deformed quantum mechanical density matrix. For convenience, further we use the deformation parameter α_{r_M} instead of $(M_0/M)^2 = (r_{\min}/r_M)^2$.*

As directly follows form formula (7), for μ = const any "shift" of the quantity r_M

$$r_M \to \widetilde{r_M} \tag{17}$$

inevitably leads to the corresponding "shift" of H_0

$$H_0 \to \widetilde{H_0} \tag{18}$$

and also of all the remaining inflationary parameters, specifically of the scale factor $a(\eta): a(\eta) \to \tilde{a}(\eta)$.

But a stationary black hole (without the absorption or emission processes) having the (SdS) metric (3), (5) is interesting from the pure academic point of view. In real physics these processes should be accounted for because they are always the case. In [10] a "shift" (17) in parameters of the black hole with SdS metrics and with small event-horizon radius in the pre-inflationary epoch have been studied taking into consideration qgcs in the case of "minimal" particle absorption by the black hole understood as a process of a minimal increment of the black-hole event horizon [30, 31]. In [30, 31] a minimal increase (ΔA)0 in the area of a black hole absorbing a classical particle of the energy E has been calculated, and the size is R given by $(\Delta A)_0 \simeq 4l_p^2(\ln 2)ER.$ In a quantum pattern we have $R \sim 2\delta x$ and $E \sim \delta p.$

However, in frames of GUP (8) it is possible to obtain a new expression for $(\Delta A)_0$ that may be considered as a qgc $(\Delta A)_{0,q}$ of $(\Delta A)_0$ to the black hole horizon area [16] (formula (27)):

$$(\Delta A)_0 \to (\Delta A)_{0,q} \approx 4l_p^2(\ln 2)\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\frac{A_0}{A}\right)\right) = 4l_p^2(\ln 2)\exp\left(-\frac{1}{2}W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right).$$

In this case explicit formulae for shifts of the basic inflationary parameters due to the corresponding quantum-gravitation effects have been found:

$$H_0 \to H_{0,q}, \quad a(\eta) \to a(\eta)_q, \quad H(\eta) \to H(\eta)_q, \quad V(\phi_0) \to V(\phi_0)_q, ..., \tag{19}$$

where $V(\phi_0) = 3m_p^2 H_0^2$ is the potential energy of inflation, $H(\eta) = a'(\eta)/a^2(\eta)$ is the dynamic Hubble parameter and so on [8, 15], where, as usually, the prime denotes differentiation with respect to η.

R e m a r k 2. *It should be noted, that for large-size black holes (i. e. for all objects adequately considered by means of a semi-classical approximation), minimal accretion of a black hole (and the corresponding above-mentioned minimal increase* $(\Delta A)_0$ *in the area of a black hole is hardly probable, as actually the real increase* ΔA *for such black holes nearly always meets the condition* $\Delta A \gg (\Delta A)_0.$ *At the same time, in the case of small-radius black holes under study this process in the early Universe is quite real.*

Black hole evaporation, qgcs and quantum-gravitational shifts of inflationary parameters. Black holes are associated with the process of radiation (Hawking evaporation). Primordial black holes are no exception. In the general case this process is considered within the scope of a semiclassical approximation (taking no account of the quantum-gravitational effects). And hence it is assumed that evaporation of a primordial black hole can be complete [8].

At the same time, taking into consideration the quantum-gravitational effects prevents complete vanishing of a black hole due to Hawking's evaporation. In the pattern proposed this is hardly the case because GUP (8) leads to the formation of a minimal (nonvanishing) Planck's remnant as a result of evaporation (10) [23]. Work [10] presents the beginning of studies into the evaporation process for black holes in the pre-inflationary epoch. Based on the general formulae ([32, p. 357]), in [10] the following expression has been obtained for the mass loss by a black hole:

$$-\frac{dM}{dt} \sim b\left(\frac{m_p}{M}\right)^2\left(\frac{m_p}{t_p}\right)N, \tag{20}$$

where $b \approx 2.59 \cdot 10^{-6}$, and N is the number of states and species of particles that are irradiated. The minus sign in the left side of the last formula means that the mass of a black hole is reduced due to evaporation, i. e. we have $\frac{dM}{dt} < 0.$ Unfortunately, this formula is not at all effective in calculations as it is difficult to estimate the number N, especially at high energies $E \simeq E_p.$

But using the terms and symbols of this paper, and also the results from [16], formula (20) for the mass loss by a black hole with regard to qgcs may be written in a more precise and constructive manner. Indeed, according to formula (45) in [16], i. e. in virtue of GUP (8), we have

$$\frac{dM}{dt} = -\frac{\gamma_1}{M^2 l_p^4}\exp\left(-2W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)\left(1 - \frac{8\gamma_2}{e\gamma_1}\left(\frac{M_0}{M}\right)^2 \exp\left(-W\left(-\frac{1}{e}\left(\frac{M_0}{M}\right)^2\right)\right)\right), \tag{21}$$

where $\gamma_1 = \frac{\pi^2}{480}, \ \gamma_2 = \frac{\pi^2}{16128}.$

The minus sign in the right side of the last formula means the same as the minus sign in the left side of the formula in (20).

Formula (21) due to (16) may be rewritten in the following way:

$$\frac{dM}{dt}=-\frac{\gamma_1}{M^2 l_p^4}\exp\left(-2W\left(-\frac{1}{e}\alpha_{rM}\right)\right)\left(1-\frac{8\gamma_2}{e\gamma_1}\alpha_{rM}\exp\left(-W\left(-\frac{1}{e}\alpha_{rM}\right)\right)\right). \tag{22}$$

The right-hand sides of formulae (21) and (22) may be expanded into a series in terms of the small parameter $e^{-1}(M_0/M)^2=e^{-1}\alpha_{rM}$ (formula (46) in [16]), which in terms of the deformation parameter α_{rM} takes the form:

$$\frac{dM}{dt}=-\frac{\gamma_1}{M^2 l_p^4}\left(1+\frac{2}{e}\alpha_{rM}+\frac{4}{e^2}\left(1-\frac{2\gamma_2}{e\gamma_1}\right)\alpha_{rM}{}^2+\ldots\right). \tag{23}$$

Integrating the last equation with respect to the selected time period Δt we can find the mass loss by a black hole for this period of time. In particular, the mass loss by a black hole with regard to qgcs by the moment of the inflation onset is equal to

$$\Delta_{\text{Evap}}M(t_M,t_{\inf l})=\int_{t_M}^{t_{\inf l}}\frac{dM}{dt}=-\int_{t_M}^{t_{\inf l}}\frac{\gamma_1}{M^2 l_p^4}\left(1+\frac{2}{e}\alpha_{rM}+\frac{4}{e^2}\left(1-\frac{2\gamma_2}{e\gamma_1}\right)\alpha_{rM}^2+\ldots\right). \tag{24}$$

Here we have $\Delta t=t_{\inf l,\text{ons}}-t_M$, where $t_{\inf l,\text{ons}}$ is the inflation onset time, and t_M is the time in which the black hole under study has been formed with regard to qgcs from formula (1).

As we have $M=r_M/2G$, the last formula may be written in the following form:

$$\Delta_{\text{Evap}}M(t_M,t_{\inf l})=\int_{t_M}^{t_{\inf l}}(2G)^{-1}\frac{dr_M}{dt}=-\int_{t_M}^{t_{\inf l}}\frac{2G\gamma_1}{r_M^2 l_p^4}\left(1+\frac{2}{e}\alpha_{rM}+\frac{4}{e^2}\left(1-\frac{2\gamma_2}{e\gamma_1}\right)\alpha_{rM}^2+\ldots\right). \tag{25}$$

Because, in accordance with the selected normalization, $G=l_p^2$, the last formula may be given only in terms of the radius r_M, of the corresponding deformation parameter α_{rM} and the known constants

$$\Delta_{\text{Evap}}M(t_M,t_{\inf l})=-\int_{t_M}^{t_{\inf l}}\frac{2\gamma_1}{Gr_M^2}\left(1+\frac{2}{e}\alpha_{rM}+\frac{4}{e^2}\left(1-\frac{2\gamma_2}{e\gamma_1}\right)\alpha_{rM}^2+\ldots\right). \tag{26}$$

By means of the last formula we can find the mass $M_{q,\text{Evap}}(t_M,t_{\inf l})$ and the radius $r_{q,\text{Evap}}(t_M,t_{\inf l})$ of a black hole after its evaporation, from the moment of its generation to the moment of the inflation onset, with regard to qgcs

$$M_{q,\text{Evap}}=M+\Delta_{\text{Evap}}M(t_M,t_{\inf l}). \tag{27}$$

Replacing $\widetilde{r_M}\to r_{M_{q,\text{Evap}}}$ in formula (17) and substituting it into (7) for μ = const, we can obtain the "shift" in formula (18)

$$H_0\to H_{0,q,\text{Evap}}=\frac{4\mu^3}{r_{M_{q,\text{Evap}}}} \tag{28}$$

and the corresponding "shifts" of all the inflationary parameters in formula (19) caused by qgcs for pbh in time from its radiation to the inflation onset.

Some direct cosmological implications. One of the most important inflationary parameters is the number of e-folds before the end of inflation, denoted as $N_e^{(\text{tot})}$, that is determined by

$$N_e^{(\text{tot})}=\ln\frac{a(t_{\text{end}})}{a(t_{\text{orig}})},$$

where $a(t_{\text{end}})$ is a value of the scale factor at the end of inflation and $a(t_{\text{orig}})$ is a value of the scale factor at the onset of inflation. The term is associated with the fact that during the period of inflation the Universe is expanded by a factor of $\exp\{N_e^{(\text{tot})}\}$.

Then we can define

$$N_{e,q}^{(\mathrm{tot})} = \ln \frac{a(t_{\mathrm{end}})_q}{a(t_{\mathrm{orig}})_q},$$

where $a(t_{\mathrm{end}})_q$ and $a(t_{\mathrm{orig}})_q$ are values of the parameters $a(t_{\mathrm{end}})$ and $a(t_{\mathrm{orig}})$ with regard to qgcs.

As directly follows from formulae (6), (7), and (17), the scale factor a with regard to qgcs for $\mu = \mathrm{const}$ is transformed up to a calculable factor in the same way as from formula (7): $r_M \propto a \propto H_0^{-1}$. In such a manner, if by the very beginning of the inflationary process a pbh with the initial mass (radius) $M_{\mathrm{orig}}, r_{M_{\mathrm{orig}}}$, due to taking into consideration qgcs, becomes a black hole with the mass $M_{\mathrm{orig},q}$ and with the radius $r_{M_{\mathrm{orig},q}}$ (e. g., as a result of the emission process (formulae (27), (28)), whereas by the end of inflation, also with regard to qgcs, the mass and the radius are $M_{\mathrm{end},q}, r_{M_{\mathrm{end},q}}$, we have

$$N_{e,q}^{(\mathrm{tot})} = \ln \frac{a(t_{\mathrm{end}})_q}{a(t_{\mathrm{orig}})_q} = \ln \frac{(r_{M_{\mathrm{end},q}} / r_{M_{\mathrm{orig},q}}) a(t_{\mathrm{end}})}{(r_{M_{\mathrm{orig},q}} / r_{M_{\mathrm{orig}}}) a(t_{\mathrm{orig}})} = \ln \frac{r_{M_{\mathrm{end},q}} r_{M_{\mathrm{orig}}}}{r^2_{M_{\mathrm{orig},q}}} + N_e^{(\mathrm{tot})}.$$

A similar shift is taken by the exponent $\exp\left\{N_e^{(\mathrm{tot})}\right\}$ as well:

$$\exp\left\{N_e^{(\mathrm{tot})}\right\} \to \exp\left\{N_{e,q}^{(\mathrm{tot})}\right\} = \frac{r_{M_{\mathrm{end},q}} r_{M_{\mathrm{orig}}}}{r^2_{M_{\mathrm{orig},q}}} \times N_e^{(\mathrm{tot})}.$$

Conclusion. In conclusion it is stated that for inflation models one can, in the explicit form, calculate the shifts of the basic parameters in these models due to the quantum-gravitational corrections of primordial black holes generated in the pre-inflation period. Besides, all cases of the expressions for the corresponding correction of *e*-foldings in an inflation model have been found.

Based on the results of this work and paper [10], the following problems may be formulated for further studies into consideration of the quantum-gravitational effects for pbhs in cosmology.

1. Which contributions are made by the shifts of the cosmological inflationary parameters, calculated (in this work and in [10]) due to qgcs for pbhs in the early Universe, into scalar and tensor cosmological perturbations arising in the process of inflation [15]?

2. How the above-mentioned contributions are related to the general approaches to the quantum-gravitational corrections for cosmological perturbations (for example, [33])?

3. As it is shown in the beginning of this paper, generation of pbhs in the early Universe has no direct relation to the inflation model, the problem is: are the above-considered methods applicable to other cosmological models, e. g., cyclic ones [34]?

The author is deeply grateful to the reviewer for a number of valuable and important comments that enable to improve the paper.

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