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TO THE QUESTIONS OF SHEMETKOV AND AGRAWAL ABOUT THE GENERALIZATIONS OF THE HYPERCENTER OF FINITE GROUPS

Abstract. A formation **F** is called a Baer – Shemetkov formation in a class **X** of groups if in any finite **X**-group the intersection of all **F**-maximal subgroups coincides with the **F**-hypercenter. It is proved that for a non-empty hereditary saturated formation **F** there exists the greatest by inclusion hereditary saturated formation BS**F** such that **F** is a Baer – Shemetkov formation in BS**F.** The connection of this result with the solution of Agrawal's (1976) and Shemetkov's (1995) questions is discussed. For the class **U** of all supersolvable groups the class BS**U** is described and the algorithm for its recognition is presented.

Keywords: finite group; supersolvable group; **F**-hypercenter; generalized hypercenter; hereditary saturated formation; Baer – Shemetkov formation

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К ВОПРОСАМ ШЕМЕТКОВА И АГРАВАЛЯ ОБ ОБОБЩЕНИЯХ ГИПЕРЦЕНТРА КОНЕЧНЫХ ГРУПП

Аннотация. Формация **F** называется формацией Бэра – Шеметкова в классе групп **X**, если в любой конечной **X**-группе пересечение **F**-максимальных подгрупп совпадает с **F**-гиперцентром. Доказано, что для непустой наследственной насыщенной формации **F** существует наибольшая по включению наследственная насыщенная формация BS**F**, в которой **F** является формацией Бэра – Шеметкова. Установлена связь данного результата с решениями вопросов Аграваля (1976) и Шеметкова (1995). Для класса **U** всех сверхразрешимых групп описан класс BS**U** и приведен алгоритм распознавания принадлежности группы данному классу.

Ключевые слова: конечная группа, сверхразрешимая группа, **F**-гиперцентр, обобщенный гиперцентр, наследственная насыщенная формация, формация Бэра – Шеметкова

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Introduction and the Main Results. All considered here groups are finite. The action of a group on its chief factors encodes the important information about the structure of this group. By restricting the action of a group on its chief factors, such important classes of groups as the classes of all nilpotent, supersolvable and solvable groups are defined. In the studying of the group's action on its chief factors and the classes of groups associated with it, the concepts of the hypercenter and its generalizations play an important role. Note that the formational generalizations of the hypercenter were developed in the works of Baer [1], Huppert [2], Shemetkov [3] and appeared in its final form in [4]. Let **F** be a class of groups. Recall [4, p. 127–128] that a chief factor H/K of a group G is called **F**-central if the semidirect product of H/K and a group $G/C_G(H/K)$ corresponding to the action of $G/C_G(H/K)$ on H/K by conjugation belongs to **F**. The **F**-hypercenter $Z_{\mathbf{F}}(G)$ of a group G is the greatest normal subgroup of G such that all chief factors of G below it are **F**-central.

The classical method for studying the hypercenter is to obtain its characterizations using various systems of subgroups. Thus, Hall [5] and Baer [6] obtained the hypercenter as the intersection of all normalizers of Sylow subgroups and maximal nilpotent subgroups, respectively. Shemetkov posed the following question on Gomel Algebraic seminar in 1995: "For what non-empty (normally) hereditary (solvably) saturated formations **F** do the intersection of all **F**-maximal subgroups coincides with the **F**-hypercenter in every group?" This question's solution for hereditary saturated formations was presented in [7]. For some families of non-saturated formations this question was solved in [8, 9]. The intersection of all **F**-maximal subgroups of a solvable group was studied in [10] for a hereditary saturated formation **F**.

D e f i n i t i o n ([9, Definition 1]). A formation **F** is called a Baer – Shemetkov formation in a class **X** of groups if in any **X**-group the intersection of all **F**-maximal subgroups coincides with the **F**-hypercenter. If **X** coincides with the class of all groups, then we say that **F** is a Baer – Shemetkov formation.

From [7] it follows that the class of all supersolvable groups **U** is not a Baer – Shemetkov formation. Nevertheless, in the symmetric group of degree 4 the intersection of all maximal supersolvable subgroups coincides with the supersolvable hypercenter. It is well known that **U** is not a Fitting formation in the general but in the class of all groups with the nilpotent derived subgroup it is. Vasil'ev and Shemetkov [11, 17.38] asked if there exists the greatest by inclusion hereditary saturated formation in which **U** is the Fitting formation. In this paper we will consider the analogous question for Baer – Shemetkov formations. If **X** is a class of groups, then we use $M(\mathbf{X})$ to denote the class of groups $G \notin \mathbf{X}$ all whose proper subgroups belong to **X**. The main result of the paper is

T h e o r e m 1. *Let* **F** *be a non-empty hereditary saturated formation. There exists the greatest by inclusion hereditary saturated formation* BS**F** *such that* **F** *is a Baer – Shemetkov formation in* BS**F**. *If F is the canonical local definition of* **F**, *then* BS**F** *is locally defined by h where h*(*p*) *is the class of all groups whose* $\mathbf{E}_{\mathbf{\Phi}}M(F(p))$-*subgroups belong to* **F**.

From this theorem the main results of [7] directly follow:

C o r o l l a r y 1 ([7, Theorems A and B]). *Let* **F** *be hereditary saturated formation with* $\pi(\mathbf{F}) \neq \varnothing$ *and F be its canonical local definition. The following statements hold*:

(1) **F** i*s a Baer – Shemetkov formation iff* $M\left(F(p)\right)\subseteq\mathbf{F}$ *for every prime p.*

(2) **F** i*s a Baer – Shemetkov formation in the class of all solvable groups* **S** *iff* $M\left(F(p)\right)\cap\mathbf{S}\subseteq\mathbf{F}$ *for every prime p.*

Using the method of arithmetic graphs developed in [12] we can describe the class BS**U**. Recall that a Schmidt group is a non-nilpotent group all whose proper subgroups are nilpotent.

T h e o r e m 2. BS**U** *coincides with the class of groups G such that* $O_{p'}(G)$ *contains a normal Sylow subgroup of every non-supersolvable subgroup H such that* $\pi(H)\subseteq\pi\left(p(p-1)\right)$ *and* $H/\Phi(H)$ *is a Schmidt group for every* $p \in \pi(G)$.

Recall [13] that the generalized center $Z_{Gn}(G)$ is a subgroup of G generated by all elements x of a group G such that $\langle x\rangle P = P\langle x\rangle$ for every Sylow subgroup P of G. Let $(Z_{Gn})_0(G) = 1$ and $(Z_{Gn})_{i+1}(G)$ be defined by $(Z_{Gn})_i(G) \subseteq (Z_{Gn})_{i+1}(G)$ and $(Z_{Gn})_{i+1}(G)/(Z_{Gn})_i(G) = Z_{Gn}(G/(Z_{Gn})_i(G))$ for $i > 0$. The terminal member of the series $1 = (Z_{Gn})_0(G) \leq (Z_{Gn})_1(G) \leq \ldots$ is denoted by $Z^*_{Gn}(G)$. According to [13, Theorem 2.10, Proposition 2.3] and [14, Chapter 1, Theorem 7.10] $Z_{\mathbf{U}}(G)\subseteq Z^*_{Gn}(G)\subseteq \mathrm{Int}_{\mathbf{U}}(G)$ for every group G. Argawal [13, p. 19] (see also [14, Chapter 1, p. 22]) asked if $Z^*_{Gn}(G)=\mathrm{Int}_{\mathbf{U}}(G)$ for every group G. The negative answer to this question was obtained in [7, 10]. With the help of Theorem 2 we can show that BS**U** is the class of groups for which the answer to Argawal's question is positive.

C o r o l l a r y 2 . *If G is a* BS**U**-*group, then* $Z^*_{Gn}(G)=\mathrm{Int}_{\mathbf{U}}(G)$.

In [15] the class of groups all whose Schmidt subgroups are supersolvable was studied. This class is a hereditary saturated formation that contains interesting and widely studied subformations w**U** [16] and v**U** [17] of groups all whose Sylow and cyclic primary subgroups respectively are **P**-subnormal. The connection of the class BS**U** with the above mentioned formations is shown in

C o r o l l a r y 3. Every BS**U**-*group is solvable. If every Schmidt subgroup of a group is supersolvable, then it is a* BS**U**-*group.*

It is natural to ask: given a group G how fast we can check if it is a BS**U**-group or not? The methods of computations in groups depend on how the group is presented (by permutations, by matrices and etc.). The most developed part of the computational group theory is the computational theory of permutation groups (see [18]). Using its methods, Theorem 2 and the definition of local formation we can find the effective algorithm to check if a permutation group is a BS**U**-group.

T h e o r e m 3. *Given a permutation group $G = \langle S\rangle$ of degree n with $|S| \le n^2$ in polynomial time in n one can check if G is a* BS**U**-*group.*

R e m a r k 1. If $|S| > n^2$, then using Sims algorithm [18, Parts 4.1 and 4.2] in polynomial time in n and $|S|$ one can find S_1 with $|S_1| \le n^2$ and $G = \langle S_1\rangle$.

R e m a r k 2. The computational theory of formations is not so developed as the computational group theory. Its main results are presented in [19–21]. Using the methods of these papers with the help of Theorem 3 and step (b) of its proof one can construct the algorithm for computing the BS**U**-residual of a group. In particular, if a permutation group $G = \langle S\rangle$ of degree n with $|S| \le n^2$ is solvable, then the BS**U**-residual of G can be computed in polynomial time by [20] or [21, Theorem 7].

R e m a r k 3. The described in the proof of this theorem algorithm was implemented in GAP [22]. The timings in the following cases were obtained using GAP 4.11.0 started with 4 GB of RAM on Intel(R) Core(TM) i5-8250U CPU @ 1.60GHz 1.80 GHz:

– All 176 groups of order 324 were checked for being BS**U**-groups in 0.85 seconds.

– All 150 groups of order 900 were checked for being BS**U**-groups in 0.9 seconds.

– All 1040 groups of order 1200 were checked for being BS**U**-groups in 5.95 seconds.

– The Dark group (see [23, Chapter IX, § 5]) has implementation in GAP in [24] of order $2^3 \cdot 3^9 \cdot 5^{24} \times \times 7 \cdot 31^8$ = 56034173979596338748931884765625000 was checked for being a BS**U**-group in 0.2 seconds.

Preliminaries. We use the standard notation and terminology of group classes theory. Recall some of them: *a class of groups* is a collection $\mathbf{X}$ of groups with the property that if $G \in \mathbf{X}$ and $H \cong G$, then $H \in \mathbf{X}$; $\mathbf{E_\Phi X}$ is the class of groups H such that H has a normal subgroup N with $N \le \Phi(H)$ and $H/N \in \mathbf{X}$; *a formation* is a class of groups $\mathbf{F}$ which is closed under taking epimorphic images (i. e. from $G \in \mathbf{F}$ and N is normal in G it follows that $G/N \in \mathbf{F}$) and subdirect products (i. e. from $G/N_1 \in \mathbf{F}$ and $G/N_2 \in \mathbf{F}$ it follows that $G/(N_1 \cap N_2) \in \mathbf{F}$); a formation $\mathbf{F} = \mathbf{E_\Phi F}$ is called *saturated*; a formation $\mathbf{F}$ is called *hereditary* if from $G \in \mathbf{F}$ and $H \le G$ it follows that $H \in \mathbf{F}$. The class $\mathbf{N}_p\mathbf{F} = (G \mid G/\mathrm{O}_p(G) \in \mathbf{F})$ is a formation for every formation $\mathbf{F}$ and prime p. We use $\pi(G)$ to denote the set of all prime divisors of $|G|$.

A function f, which assigns to every prime a formation, is called *a formation function*. Recall [23, Chapter IV, Definitions 3.1] that a formation $\mathbf{F}$ is called *local* if

$$\mathbf{F} = \left(G \mid G/C_G(H/K) \in f(p) \text{ for every } p \in \pi(H/K) \text{ and every chief factor } H/K \text{ of } G\right)$$

for some formation function f. In this case f is called *a local definition* of $\mathbf{F}$. According to Gashüts – Lubeseder – Schmid theorem [23, Chapter IV, Theorem 4.6] a formation is non-empty saturated iff it is local. Recall [23, Chapter IV, Theorem 3.7] that if $\mathbf{F}$ is a local formation, then there exists a unique local definition F of $\mathbf{F}$, such that F is *integrated* (i. e., $F(p) \subseteq \mathbf{F}$ for all prime p) and *full* (i. e., $F(p) = \mathbf{N}_pF(p)$ for all prime p). Such definition F is called *the canonical local definition* of $\mathbf{F}$.

Recall that a Schmidt (p, q)-group is a Schmidt $\{p, q\}$-group with a normal Sylow p-subgroup. The N-critical graph $\Gamma_{Nc}(G)$ [12, Definition 1.3] of a group G is a directed graph whose vertices are prime divisors of $|G|$ and (p, q) is an edge of $\Gamma_{Nc}(G)$ iff G contains a Schmidt (p, q)-subgroup. The N-critical graph of class of groups $\mathbf{X}$ [12, Definition 3.1] is defined by $\Gamma_{Nc}(\mathbf{X}) = \cup_{G \in \mathbf{X}}\Gamma_{Nc}(G)$. For a graph Γ we use $E(\Gamma)$ to denote the set of all its edges.

Proofs of the Main Results. The proof of Theorem 1 is based on the following two lemmas.

L e m m a 1. *Let F be the canonical local definition of a local formation* $\mathbf{F}$. *Then the class* $\mathbf{X}_p$ *of all groups whose* $\mathbf{E_\Phi}M(F(p))$*-subgroups belong to* $\mathbf{F}$ *is a hereditary formation and* $\mathbf{X}_p = \mathbf{N}_p\mathbf{X}_p$ *for every prime p.*

P r o o f. Let p be a prime and $\mathbf{X}$ be the class of all groups whose $\mathbf{E}_\Phi M(F(p))$-subgroups belong to $\mathbf{F}$.

(a) $\mathbf{X}$ *is closed under taking quotients.*

Let G be an $\mathbf{X}$-group, N be a normal subgroup of G, H/N be an $\mathbf{E}_\Phi M(F(p))$-subgroup of G/N and K be a minimal supplement to N in H. Then $K \cap N \leq \Phi(K)$ by [23, Chapter A, Theorem 9.2(c)]. Therefore $H/N = KN/N \cong K/(K \cap N) \in \mathbf{E}_\Phi M(F(p))$. Recall [23, Chapter A, Theorem 9.2(e)] that $\Phi(T/R) = \Phi(T)/R$ for any group T and its normal subgroup R with $R \leq \Phi(T)$. It means that $K \in \mathbf{E}_\Phi M(F(p))$. From our assumption it follows that $K \in \mathbf{F}$. Since $\mathbf{F}$ is a formation, $H/N \cong K/(K \cap N) \in \mathbf{F}$. Thus, $G/N \in \mathbf{X}$ and hence $\mathbf{X}$ is closed under taking quotients.

(b) $\mathbf{X}$ *is closed under taking subdirect products.*

Let A and B be normal subgroups of a group G with $A \cap B \cong 1$ such that G/A and G/B are $\mathbf{X}$-groups and H be an $\mathbf{E}_\Phi M(F(p))$-subgroup of G. Recall [23, Chapter A, Theorem 9.2(e)] that $\Phi(T)R/R \leq \Phi(T/R)$ for any group T and its normal subgroup R. Therefore, $Q = (H/(H \cap A))/\Phi(H/(H \cap A))$ is isomorphic to a quotient group of a group from $M(F(p)) \subseteq \mathbf{E}_\Phi M(F(p))$. It means that either $Q \in F(p) \subseteq \mathbf{F}$ or $Q \in M(F(p))$. Since $\mathbf{F}$ is a saturated formation, $H/(H \cap A) \in \mathbf{F}$ in the first case. In the second case $H/(H \cap A) \in \mathbf{E}_\Phi M(F(p))$. From $H/(H \cap A) \cong HA/A \leq G/A \in \mathbf{X}$ it follows that $H/(H \cap A) \in \mathbf{F}$ by the definition of $\mathbf{X}$. Therefore, $H/(H \cap A) \in \mathbf{F}$ in both cases. By analogy $H/(H \cap B) \in \mathbf{F}$. Now $H \cong H/((H \cap A) \cap (H \cap B)) \in \mathbf{F}$. Thus, $G \in \mathbf{X}$ and, hence, $\mathbf{X}$ is closed under taking subdirect products.

(c) $\mathbf{X}$ *is a hereditary formation.*

From (a) and (b) it follows that $\mathbf{X}$ is a formation. Let H be a subgroup of an $\mathbf{X}$-group G and K be an $\mathbf{E}_\Phi M(F(p))$-subgroup of H. From $K \leq G$ it follows that $K \in \mathbf{F}$. Hence $H \in \mathbf{X}$. It means that $\mathbf{X}$ is a hereditary formation.

(d) $\mathbf{X} = \mathbf{N}_p\mathbf{X}$.

Assume that $\mathbf{X} \neq \mathbf{N}_p\mathbf{X}$. It means that $\mathbf{X} \subset \mathbf{N}_p\mathbf{X}$. Let G be a minimal order group from $\mathbf{N}_p\mathbf{X} \setminus \mathbf{X}$. Since $\mathbf{X}$ is a hereditary formation, we see that $\mathbf{N}_p\mathbf{X}$ is a hereditary formation too. Therefore, all proper subgroups of G belong to $\mathbf{X}$, G has the unique minimal normal subgroup N, $G/N \in \mathbf{X}$ and N is a p-group. If all $\mathbf{E}_\Phi M(F(p))$-subgroups of G are proper, then they belong to $\mathbf{X}$ and, hence, to $\mathbf{F}$. Therefore, $G \in \mathbf{X}$, the contradiction. It means that $G \in \mathbf{E}_\Phi M(F(p))$ and $G \notin \mathbf{F}$.

If $N \leq \Phi(G)$, then G/N is either an $\mathbf{E}_\Phi M(F(p))$-group or $\mathbf{E}_\Phi F(p)$-group. In the first case $G/N \in \mathbf{F}$ by the definition of $\mathbf{X}$ and in the second case $G/N \in \mathbf{F}$ by $\mathbf{E}_\Phi F(p) \subseteq \mathbf{E}_\Phi \mathbf{F} = \mathbf{F}$. Since $\mathbf{F}$ is saturated, we see that $G \in \mathbf{F}$, the contradiction. Therefore, $\Phi(G) \cong 1$. Hence $G \in M(F(p))$. Note that in this case N has the complement M in G. Now $G/N = MN/N \cong M \in F(p)$. Hence $G \in \mathbf{N}_pF(p) = F(p) \subseteq \mathbf{F}$, the final contradiction. Thus, $\mathbf{X} = \mathbf{N}_p\mathbf{X}$. □

L e m m a 2. *Let* $\mathbf{H}$ *and* $\mathbf{F}$ *be non-empty hereditary saturated formations locally defined by* H *and* F *respectively where* F *is the canonical local definition of* $\mathbf{F}$ *and* H *is a full local definition of* $\mathbf{H}$ *such that* $H(p)$ *is hereditary for all prime* p. *The following statements hold*:

(1) *If* $M(F(p)) \cap H(p) \subseteq \mathbf{F}$ *for every prime* p, *then* $\mathbf{F}$ *is a Baer – Shemetkov formation in* $\mathbf{H}$.

(2) *Assume that* H *is a canonical local definition of* $\mathbf{H}$ *and* $\mathbf{F}$ *is a Baer – Shemetkov formation in* $\mathbf{H}$. *Then* $M(F(p)) \cap H(p) \subseteq \mathbf{F}$ *for every prime* p.

P r o o f. Note that $F(p)$ is a hereditary formation by [23, Chapter IV, Proposition 3.16] for every prime p.

(1) *Assume that* $M(F(p)) \cap H(p) \subseteq \mathbf{F}$ *for every prime* p.

Suppose that there exists an $\mathbf{H}$-group G with $\mathrm{Int}_{\mathbf{F}}(G) \neq \mathrm{Z}_{\mathbf{F}}(G)$. Without loss of generality, we may assume that G is a minimal order group with such property. Note that $\mathrm{Z}_{\mathbf{F}}(H) \leq \mathrm{Int}_{\mathbf{F}}(H)$ for any group H by [7, Theorem C(h)]. From $1 \in \mathbf{F} \cap \mathbf{H}$ it follows that $\mathrm{Z}_{\mathbf{F}}(G) < \mathrm{Int}_{\mathbf{F}}(G) \not\leq (1)$.

Let N be a minimal normal subgroup of G with $N \leq \mathrm{Int}_{\mathbf{F}}(G)$ and M be an $\mathbf{F}$-maximal subgroup of G. If N is abelian, then N is a p-group for some prime p. Let $1 = M_0 \leq M_1 \leq \ldots \leq M_m = N$ be a part of chief series of M. Then $M/C_M(M_i/M_{i-1}) \in F(p)$. Let $C = \cap_{i=1}^m C_M(M_i/M_{i-1})$. Now $C/C_M(N)$ is a p-group by [23, Chapter A, Corollary 12.4(a)]. Since $F(p)$ is a formation, $M/C \in F(p)$. Thus, $M/C_M(N) \in \mathbf{N}_pF(p) = F(p)$. If N is non-abelian, then N is the direct product of minimal normal subgroups M_i, $i = 1,\ldots, m$, of M by [23, Chapter A, Lemma 4.14]. Note that $C_M(N) = \cap_{i=1}^m C_M(M_i)$. Since $F(p)$ is a formation, $M/C_M(N) \in F(p)$ for all

$p \in \pi(N)$. Hence $MC_G(N)/C_G(N) \cong M/C_M(N) \in F(p)$ for all $p \in \pi(N)$. Since $G \in \mathbf{H}$ and $\mathbf{H}$ is a local formation, we see that $G/C_G(N) \in H(p)$ for all $p \in \pi(N)$.

Assume that $G/C_G(N) \notin F(p)$ for some $p \in \pi(N)$. Therefore, $G/C_G(N)$ contains a minimal non-$F(p)$-subgroup $A/C_G(N)$. Note that $A/C_G(N) \in H(p)$. By our initial assumption $A/C_G(N) \in \mathbf{F}$. Hence $A/C_G(N)$ is contained in some $\mathbf{F}$-maximal subgroup $B/C_G(N)$ of $G/C_G(N)$. From [25, Chapter 1, Lemma 5.7(ii)] it follows that there exists an $\mathbf{F}$-maximal subgroup C of G such that $CC_G(N)/C_G(N) = B/C_G(N)$. Therefore, $CC_G(N)/C_G(N) \in F(p)$. Now $A/C_G(N) \in F(p)$, the contradiction.

Thus, $G/C_G(N) \in F(p)$ for all $p \in \pi(N)$. It means that $N \le Z_{\mathbf{F}}(G)$ by [25, Chapter 1, Proposition 1.15(1)]. Note that $Z_{\mathbf{F}}(G)/N = Z_{\mathbf{F}}(G/N)$ by [25, Chapter 1, Theorem 2.6(f)] and $\mathrm{Int}_{\mathbf{F}}(G)/N = \mathrm{Int}_{\mathbf{F}}(G/N)$ by [7, Theorem C(e)]. From the choice of G it follows that $\mathrm{Int}_{\mathbf{F}}(G/N) = Z_{\mathbf{F}}(G/N)$. Therefore,

$$\mathrm{Int}_{\mathbf{F}}(G)/N = \mathrm{Int}_{\mathbf{F}}(G/N) = Z_{\mathbf{F}}(G/N) = Z_{\mathbf{F}}(G)/N.$$

Thus, $\mathrm{Int}_{\mathbf{F}}(G) = Z_{\mathbf{F}}(G)$, the final contradiction. Hence $\mathbf{F}$ is a Baer – Shemetkov formation in $\mathbf{H}$.

(2) *Assume that H is the canonical local definition of* $\mathbf{H}$ *and* $\mathbf{F}$ *is a Baer – Shemetkov formation in* $\mathbf{H}$.

Note that $H(p)$ is a hereditary formation by [23, Chapter IV, Proposition 3.16] for every prime p and hence we have no contradiction with the initial assumption of lemma.

Suppose that for some prime p there exists an $H(p)$-group $G \notin \mathbf{F}$ such that all its proper subgroups are $F(p)$-groups. Without loss of generality, we may assume that G is a minimal order group with such property.

Let N be a minimal normal subgroup of G. Note that $G/N \in H(p)$ and all its proper subgroups belong to $F(p)$. Hence $G/N \in \mathbf{F}$. Since $\mathbf{F}$ is a saturated formation, we see that G has the unique minimal normal subgroup and $\Phi(G) \cong 1$. Hence there exists a maximal subgroup M of G with $MN = G$. Note that $M \in F(p)$. Therefore, $G/N = MN/N \cong M/(M \cap N) \in F(p)$.

Assume that N is a p-group. Then $G \in \mathbf{N}_pF(p) = F(p) \subseteq \mathbf{F}$, the contradiction. It means that $O_p(G) \cong 1$.

From [23, Chapter B, Theorem 10.3] it follows that G has a faithful irreducible module V over GF(p). Let T be the semidirect product of V and G corresponding to the action of G on V as a G-module over GF(p). Now $T \in \mathbf{N}_pH(p) = H(p) \subseteq \mathbf{H}$.

Let L be an $\mathbf{F}$-maximal subgroup of T. From $G \notin \mathbf{F}$ it follows that $LV < T$. If R is a maximal subgroup of T with $V \le R$, then R/V is isomorphic to a proper subgroup of G. It means that $R/V \in F(p)$. Therefore, $R \in \mathbf{N}_pF(p) = F(p) \subseteq \mathbf{F}$. Thus, the sets of all $\mathbf{F}$-maximal subgroup of T and of all maximal subgroups M of T with $V \le M$ coincide. Now $V \le \mathrm{Int}_{\mathbf{F}}(T) = Z_{\mathbf{F}}(T)$. Hence $T/C_T(V) \cong G \in F(p) \subseteq \mathbf{F}$ by [25, Chapter 1, Proposition 1.15(1)], the final contradiction. It means that every minimal non-$F(p)$-group from $H(p)$ belongs to $\mathbf{F}$ for every prime p. □

P r o o f o f T h e o r e m 1. For a prime p let $h(p)$ be the class of all groups whose $\mathbf{E}_{\mathbf{\Phi}}M(F(p))$-subgroups belong to $\mathbf{F}$. From Lemma 1 it follows that $h(p) = \mathbf{N}_ph(p)$ is a hereditary formation. Let BS$\mathbf{F}$ be a local formation locally defined by h. Hence h is a full local definition of BS$\mathbf{F}$. Note that BS$\mathbf{F}$ is a hereditary saturated formation by [23, Chapter IV, Proposition 3.14 and Theorem 4.6]. Now $\mathbf{F}$ is a Baer – Shemetkov formation in BS$\mathbf{F}$ by (1) of Lemma 2. Assume that $\mathbf{K}$ is a non-empty hereditary saturated formation such that $\mathbf{F}$ is a Baer – Shemetkov formation in $\mathbf{K}$. Let K be the canonical local definition of $\mathbf{K}$. Then every minimal non-$F(p)$-group from $K(p)$ belongs to $\mathbf{F}$ for every prime p by (2) of Lemma 2. Since $K(p)$ is a formation and $\mathbf{F}$ is saturated, every $\mathbf{E}_{\mathbf{\Phi}}M(F(p))$-group from $K(p)$ belongs to $\mathbf{F}$ for every prime p. Note that $K(p)$ is a hereditary formation for all prime p. Hence every $\mathbf{E}_{\mathbf{\Phi}}M(F(p))$-subgroup from a $K(p)$-group belongs to $\mathbf{F}$. It means that $K(p) \subseteq h(p)$ for all prime p. Thus, $\mathbf{K} \subseteq$ BS$\mathbf{F}$. It means that BS$\mathbf{F}$ is the greatest by inclusion hereditary saturated formation such that $\mathbf{F}$ is a Baer – Shemetkov formation in BS$\mathbf{F}$. □

P r o o f o f T h e o r e m 2. Recall that $\mathbf{A}(n)$ denotes the class of all abelian groups of exponent, dividing n, and the canonical local definition of $\mathbf{U}$ is F where $F(p) = \mathbf{N}_p\mathbf{A}(p-1)$. Let p be a prime and $G \in M(\mathbf{N}_p\mathbf{A}(p-1))$. Since $\mathbf{U}$ is a saturated formation, if $G \in \mathbf{U}$, then $\mathbf{E}_{\mathbf{\Phi}}(G) \subseteq \mathbf{U}$. From Theorem 1 it follows that BS$\mathbf{U}$ is locally defined by h, where $h(p)$ is the class of all groups whose $\mathbf{E}_{\mathbf{\Phi}}M(F(p))$-subgroups belong to $\mathbf{U}$. Therefore, to describe the local definition of BS$\mathbf{U}$ we need only to consider non-supersolvable G.

Let $p \in \pi(G)$. Assume that G is not a Schmidt group. Then $\Gamma_{Nc}(G)$ is the join of $\Gamma_{Nc}(H)$ where H runs through all proper subgroups of G. From $H \in \mathbf{N}_p\mathbf{A}(p-1)$ it follows that any edge of $\Gamma_{Nc}(H)$ can start only from p. It means that $\Gamma_{Nc}(G)$ has no cycles. From [12, Theorem 6.2(b) and its proof] it follows that G is a Sylow tower group and has a normal Sylow p-subgroup P. By Schur – Zassenhaus theorem P has a complement T in G. From $T < G$ it follows that $T \in \mathbf{A}(p-1)$. Therefore, $G \in \mathbf{N}_p\mathbf{A}(p-1)$, a contradiction.

Thus, if non-supersolvable $G \in M(\mathbf{N}_p\mathbf{A}(p-1))$ and $p \in \pi(G)$, then G is a Schmidt $\{p, q\}$-group. Note that in this case $Z_q \in \mathbf{N}_p\mathbf{A}(p-1)$. Hence $q \in \pi(p-1)$. If G has a normal Sylow p-subgroup, then G is supersolvable, a contradiction. Thus, G is a Schmidt (q, p)-group for some $q \in \pi(p-1)$. From the other hand a Schmidt (q, p)-group with $q \in \pi(p-1)$ and the trivial Frattini subgroup does not belong to $\mathbf{N}_p\mathbf{A}(p-1)$ but all its proper subgroups (abelian groups of exponent 1, p or q) belong to $\mathbf{N}_p\mathbf{A}(p-1)$.

Let $p \notin \pi(G)$. Then $G \in M(\mathbf{A}(p-1))$. If the exponent of G does not divide $p-1$, then G is a cyclic primary group and, hence, supersolvable, a contradiction. Thus, G is a Miller – Moreno group. Then G is either a primary group or a Schmidt group. Since G is not supersolvable, we see that G is a Schmidt (r, q)-group for some $r, q \in \pi(p-1)$ with $q \notin \pi(r-1)$.

Therefore, every non-supersolvable group from $M(\mathbf{N}_p\mathbf{A}(p-1))$ is a Schmidt group. From the other hand for every $r, q \in \pi(p(p-1))$ with $q \notin \pi(r-1)$ a Schmidt (r, q)-group with the trivial Frattini subgroup belongs to $M(\mathbf{N}_p\mathbf{A}(p-1))$. Thus, $\mathbf{E_\Phi} M(\mathbf{N}_p\mathbf{A}(p-1))$ contains all non-supersolvable Schmidt $\pi(p(p-1))$-groups.

Let $g(p) = (G \mid (r,q) \notin E(\Gamma_{Nc}(G))$ for every $r, q \in \pi(p(p-1))$ with $q \notin \pi(r-1))$ and $h(p)$ be the class of all groups whose $\mathbf{E_\Phi} M(\mathbf{N}_p\mathbf{A}(p-1))$-subgroups belong to $\mathbf{U}$. According to [12, Theorem 3.5(2)] $g(p)$ is a hereditary formation and every minimal non-$g(p)$-group is a Schmidt (r, q)-group for some $(r,q) \notin E(\Gamma_{Nc}(g(p)))$.

Assume that $g(p) \setminus h(p) \neq \varnothing$. Let G be a minimal order group from $g(p) \setminus h(p)$. Since $h(p)$ and $g(p)$ are hereditary formations, $G \in M(h(p))$. From the definition of $h(p)$ it follows that G is a non-supersolvable group from $\mathbf{E_\Phi} M(\mathbf{N}_p\mathbf{A}(p-1))$. It means that $G/\Phi(G)$ is a Schmidt (r, q)-group for some r, $q \in \pi(p(p-1))$ with $q \notin \pi(r-1)$. Thus, $G \notin g(p)$ by the definition of $g(p)$, a contradiction. Thus, $g(p) \subseteq h(p)$ for every prime p.

Assume that $h(p) \setminus g(p) \neq \varnothing$. Let G be a minimal order group from $h(p) \setminus g(p)$. Since $h(p)$ and $g(p)$ are hereditary formations, $G \in M(g(p))$. Hence G is a Schmidt (r, q)-group for some $(r,q) \notin E(\Gamma_{Nc}(g(p)))$. It means that $h(p)$ contains a Schmidt (r, q)-group with r, $q \in \pi(p(p-1))$, $q \notin \pi(r-1)$ and the trivial Frattini subgroup. Note that this group is non-supersolvable and belongs to $M(\mathbf{N}_p\mathbf{A}(p-1))$, a contradiction with the definition of $h(p)$. Hence $h(p) \subseteq g(p)$ for every prime p. Thus, $g(p) = h(p)$ for every prime p.

Since BS**U** is locally defined by its full definition g, a group $G \in$ BS**U** iff $G/\mathrm{O}_{p'}(G) \in g(p)$ for all $p \in \pi(G)$. Let N be a normal subgroup of G, H/N be a Schmidt group of G/N and K be a minimal supplement to N in H. Then $K \cap N \leq \Phi(K)$ by [23, Chapter A, Theorem 9.2(c)]. From $H/N \cong K/(K \cap N)$ it follows that there exists a subgroup K of G such that $K/\Phi(K)$ is a Schmidt group and $KN/N = H/N$. Therefore, G/N does not contain Schmidt (r, q)-subgroups iff N contains the nilpotent residual of every subgroup K such that $K/\Phi(K)$ is a Schmidt (r, q)-group. Since the class of all r-closed groups is saturated, such subgroups K are r-closed. Note that the nilpotent residual of $K/\Phi(K)$ is its unique minimal normal r-subgroup $R/\Phi(K)$. Hence from [23, Chapter A, Theorem 9.13] it follows that every K-composition series of the Sylow r-subgroup of K (starting from 1) must have as the final factor a factor K-isomorphic to $R/\Phi(K)$. Thus, the nilpotent residual of K coincides with its normal Sylow r-subgroup. Therefore, $G/\mathrm{O}_{p'}(G) \in g(p)$ iff $\mathrm{O}_{p'}(G)$ contains a normal Sylow subgroup of every non-supersolvable subgroup H of G such that $\pi(H) \subseteq \pi(p(p-1))$ and $H/\Phi(H)$ is a Schmidt subgroup. □

P r o o f o f C o r o l l a r y 3. Assume that BS**U** contains a non-solvable group. Since BS**U** is a hereditary formation, we see that it contains a simple non-abelian group G. If the lengths of every cycle of $\Gamma_{Nc}(G)$ which contains 2 is greater than 3, then G is solvable by [12, Theorem 6.2(a)], the contradiction. Hence some cycle of $\Gamma_{Nc}(G)$ contains 2, i. e. $(2,q) \in E(\Gamma_{Nc}(G))$. It means that $q \in \pi(G)$ and $G \cong$ $\cong G/\mathrm{O}_{q'}(G) \in h(q) = g(q)$ (see the proof of Theorem 2). Note that $2, q \in \pi(q(q-1))$ and a Schmidt $(2, q)$-group is non-supersolvable. Thus, $(2,q) \notin E(\Gamma_{Nc}(G))$, the contradiction. It means that every BS**U**-group is solvable.

If every Schmidt subgroup of G is supersolvable, then from Theorem 2 it directly follows that G is a BS**U**-group. □

We need the following results in the proof of Theorem 3.

L e m m a 3. *Let $G = PQ$ where P is a Sylow p-subgroup and Q is a Sylow q-subgroup of G and* $\mathbf{F}$ *be a formation of all p-closed groups. Then $G^{\mathbf{F}}$ is a normal closure in G of a Sylow q-subgroup of* $[P, Q]$.

P r o o f. Let N be a normal subgroup of G such that G/N is p-closed. Then PN/N is a normal Sylow p-subgroup of G/N. Hence $[P, Q]N/N \subseteq PN/N$. Therefore, N contains a Sylow q-subgroup of $[P, Q]$ and, hence, its normal closure in G.

Assume now that N is a normal closure in G of a Sylow q-subgroup of $[P, Q]$. Hence $N \subseteq G^{\mathbf{F}}$. Note that $P[P, Q]$ is a normal subgroup of G by [23, Chapter A, Lemma 7.4(h)], $N \subseteq P[P, Q]$ and $P[P, Q]/N$ is a Sylow p-subgroup of G/N. Hence $P[P, Q]/N$ is a normal Sylow p-subgroup of G/N. Thus, $N = G^{\mathbf{F}}$. □

L e m m a 4. *Given a group $G = \langle S\rangle \leq S_n$ with $|S| \leq n^2$ each of the following constructions can be carried out in polynomial time*:

(1) [26, Main Theorem] *given $p \in \pi(G)$, find a Sylow p-subgroup P of G;*

(2) [18, p. 49, (g)(i)] *given $T \subseteq G$, find the normal closure $\langle T^G\rangle$;*

(3) [26, Theorem A.2] *if G is solvable*, *given $\pi \subseteq \pi(G)$, find a Hall π-subgroup H of G;*

(4) [18, p. 49, (g)(iii)] *test G for solvability*;

(5) [18, p. 49, (k)(ii)] *find a chief series for G;*

(6) [18, p. 49, (c)] *find* $|G|$.

P r o o f o f T h e o r e m 3. In this section we will write all algorithms in the notation of computer algebra system GAP [22]. The correctness of these algorithms does not depend on the group's presentation. Nevertheless, for at least permutation groups (of degree n) they work in polynomial time in n.

(a) The p-closed residual of a $\{p, q\}$-group $G \leq S_n$ can be computed in polynomial time.

According to Lemma 4(1) a Sylow p-subgroup $P = \langle X\rangle$ and a Sylow q-subgroup $Q = \langle Y\rangle$ of G can be computed in polynomial time. Note that $[P, Q] = \langle [x, y] \mid x \in X, y \in Y\rangle^{\langle X,Y\rangle}$. By Lemma 4(2) we can compute $[P, Q]$ in polynomial time. Hence its Sylow q-subgroup R can be computed in polynomial time by Lemma 4(1). Thus, R^G can be computed in polynomial time by Lemma 4(2). According to Lemma 3 it is the required residual.

```
ResPclosed:=function(G, p, q)
return NormalClosure(G, SylowSubgroup(CommutatorSubgroup(SylowSubgroup(G, p), SylowSubgroup(G, q)), q));
end;;
```

(b) Given a solvable group $G \leq S_n$ and a set of primes $\pi \subseteq \pi(G)$ the residual for a class of groups all whose Schmidt π-subgroups are supersolvable can be computed in polynomial time.

Note that a Schmidt (p, q)-group is supersolvable iff $q \in \pi(p - 1)$. If every Schmidt (p, q)-subgroup with $q \notin \pi(p - 1)$ of a solvable group is supersolvable (i. e. there no such subgroups), then (q, p) is the only possible edge of the N-critical graph of every Hall $\{p, q\}$-subgroup. It means that every Hall $\{p, q\}$-subgroup of a group is q-closed by [12, Theorem 6.2(2)]. From the other hand, if every Hall $\{p, q\}$-subgroup of a solvable group with $q \notin \pi(p - 1)$ is q-closed, then it has no Schmidt (p, q)-subgroups, i. e. every its Schmidt (p, q)-subgroup is supersolvable. It means that the required residual is the normal closure of q-closed residuals of every Hall $\{p, q\}$-subgroup with $\{p, q\} \subseteq \pi$ and $q \notin \pi(p - 1)$.

Since G is a permutation group of degree n, $p \leq n$ for every prime divisor p of $|G|$. Note that a Hall $\{p, q\}$-subgroup of G and its p-closed residual can be found in polynomial time for every $\{p, q\} \subseteq \pi(G)$ by Lemma 4(3) and (a). Thus, for every $\{p, q\} \subseteq \pi$ we can compute p-closed or/and q-closed residuals of a Hall $\{p, q\}$-subgroup of G in polynomial time. Note that the normal closure of computed residuals can be found in polynomial time by Lemma 4(2).

```
ResSupSchPi:=function(G, pi)
local a, b, S;
S:=[];
for a in pi do
  for b in pi do
    if (a<>b) and (not a in PrimeDivisors(b-1)) then
```

```
        Append(S,GeneratorsOfGroup(ResPclosed(HallSubgroup(G, [a, b]),a,b)));
      fi;
    od;
  od;
  return NormalClosure(G, Subgroup(G, S));
  end;;
```

(c) Given $G \le S_n$ in polynomial time one can check if G is a BS**U**-*group.*

According to Corollary 2.2, if G is not solvable, then it is not a BS**U**-group. The check for solvability can be done in polynomial time by Lemma 4(4). Now we can compute chief series of G in polynomial time by Lemma 4(5). According to (b) we can compute $G^{g(p)}$ in polynomial time. Note that $G/C_G(H/K) \in g(p)$ iff $[H, G^{g(p)}] \subseteq K$ iff $|\langle [H, G^{g(p)}], K\rangle| = |K|$. Hence by analogy with (a) we can check this condition in polynomial time by Lemma 4(6). Now (c) follows from the fact that every chain of subgroups in S_n (and hence in G) has at most $2n$ terms [27].

```
IsBSU:=function(G)
local a,l, S, p, p0, p1;
if not IsSolvable(G) then return false; fi;
S:=ChiefSeries(G); ##S[1] = G
l:=Length(S);
p0:=PrimeDivisors(Order(G));
for a in [1..(l-1)] do
  p:=PrimeDivisors(Order(S[a])/Order(S[a+1]))[1];
  p1:=[]; Append(p1, PrimeDivisors(p-1)); Add(p1, p); p1:=Set(p1); IntersectSet(p1,p0);
  if not IsSubgroup(S[a+1], CommutatorSubgroup(S[a], ResSupSchPi (G, p1))) then
    return false;
  fi;
od;
return true;
end;; □
```

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