ISSN 1561-2430 (Print)
ISSN 2524-2415 (Online)
UDC 004.9:656.6
https://doi.org/10.29235/1561-2430-2025-61-1-63-73

Received 12.08.2024
Поступила в редакцию 12.08.2024

Denis S. Sarazhinsky[1], Denis V. Kapski[1,2]

[1]Belarusian National Technical University, Minsk, Republic of Belarus
[2]Academy of Public Administration under the President of the Republic of Belarus, Minsk, Republic of Belarus

AN ANALYTICAL MODEL OF PEDESTRIAN DELAY FOR PEDESTRIAN-ACTUATED CYCLE LENGTH PRESERVING SIGNAL TRAFFIC CONTROL

Abstract. This paper considers the problem of developing a rigorous analytical model for estimating pedestrian delays at a signalized intersection when the pedestrian traffic at this intersection is controlled by a "smart" algorithm that operates according to the following principle: if "the pedestrian call button has not been pressed", skip the pedestrian service interval reserved by the control scheme and pass the unused time to conflicting road users (thereby preserving the length of the control cycle), otherwise activate the reserved interval and serve the pedestrians. Under the assumption of a Poisson process of arrivals, a rigorous development of the corresponding model and its comparison with the existing best-known and used analogue is performed based on the apparatus of probability theory. By means of a computational experiment it is shown that the proposed model is much more accurate and correct than this analogue. Finally, a primary analysis of the model is performed, with results allowing to assess the appropriateness of implementing such a control algorithm in terms of the significant increase in individual pedestrian delays.

Keywords: signal traffic control algorithms, pedestrian-actuated control, pedestrian delay, analytical model, simulation-based validation, intelligent transportation system

For citation. Sarazhinsky D. S., Kapski D. V. An analytical model of pedestrian delay for pedestrian-actuated cycle length preserving signal traffic control. *Vestsi Natsyyanal'nai akademii navuk Belarusi. Seryya fizika-matematychnykh navuk = Proceedings of the National Academy of Sciences of Belarus. Physics and Mathematics series*, 2025, vol. 61, no. 1, pp. 63–73. https://doi.org/10.29235/1561-2430-2025-61-1-63-73

Д. С. Саражинский[1], Д. В. Капский[1,2]

[1]Белорусский национальный технический университет, Минск, Республика Беларусь
[2]Академия управления при Президенте Республики Беларусь, Минск, Республика Беларусь

АНАЛИТИЧЕСКАЯ МОДЕЛЬ ЗАДЕРЖЕК ПЕШЕХОДОВ ДЛЯ АЛГОРИТМА СВЕТОФОРНОГО РЕГУЛИРОВАНИЯ С КНОПКОЙ ВЫЗОВА ПЕШЕХОДА

Аннотация. Рассматривается проблема построения строгой аналитической модели для оценки задержек пешеходов на регулируемом перекрестке в случае, когда движение пешеходов на этом перекрестке управляется с помощью «умного» алгоритма, работающего по принципу: если «не была нажата кнопка вызова пешехода», пропустить зарезервированный схемой регулирования интервал обслуживания пешеходов и передавать неиспользованное время конфликтующим участникам движения (тем самым сохраняя длительность цикла регулирования), в противном случае активировать зарезервированный интервал и обслужить пешеходов. В предположении пуассоновского потока прибытий, на базе аппарата теории вероятностей выполняется строгое построение соответствующей модели и ее сравнение с существующим наиболее известным и используемым аналогом. С помощью вычислительного эксперимента показывается, что предложенная модель намного более точная и корректная, чем этот аналог. В завершение выполняется первичный анализ модели с результатами, позволяющими оценивать целесообразность введения такого алгоритма управления с точки зрения значительности роста индивидуальных задержек пешеходов.

Ключевые слова: алгоритмы светофорного регулирования, управление с кнопкой вызова пешехода, задержка пешеходов, аналитическая модель, валидация на базе имитационного эксперимента, интеллектуальные транспортные системы

Для цитирования. Саражинский, Д. С. Аналитическая модель задержек пешеходов для алгоритма светофорного регулирования с кнопкой вызова пешехода / Д. С. Саражинский, Д. В. Капский // Весці Нацыянальнай акадэміі навук Беларусі. Серыя фізіка-матэматычных навук. – 2025. – Т. 61, № 1. – С. 63–73. https://doi.org/10.29235/1561-2430-2025-61-1-63-73

Introduction. Traffic signal control is one of the key elements of urban traffic management, and is the "heart" of intelligent transportation systems. It is therefore crucial to be able to estimate effectively the quality indicator values for such control on a theoretical level. In view of the growing emphasis on the quality of the pedestrian traffic service in recent decades [1], there is an increasing necessity to obtain such estimates for pedestrians (and not merely for vehicles, as was previously the case). One of the most fundamental indicators of the quality of the control is a so-called total pedestrian delay MD_T, which represents the mathematically expected number of person-seconds lost to society over a given observation period T due to the delay of pedestrians at a prohibited traffic signal. Another related quantity is a relative (or average) delay Md_T, which is the total delay divided by the mathematically expected number of observed pedestrians [2, 3]. In general, two approaches to obtaining estimates for these quantities can be distinguished: analytical (based on the development of analytical models, presented in the form of closed mathematical formulae) and computational (based on the development of computational models and computational experiments) [4–6]. The analytical approach has the distinct advantage of enabling the prediction of the behavior/properties of the modelled object or phenomenon in a general form (through the mathematical analysis of the formulae). Furthermore, it is frequently more straightforward to utilize this approach to obtain the results of a given accuracy, thereby conferring greater efficiency (in terms of the result/cost ratio) than the computational one [7]. However, the capacity to develop an analytical model is constrained by the degree of complexity inherent to the object or phenomenon under examination. In particular, the feasibility of deriving analytical estimates for the indicators is largely contingent upon the complexity of the signal traffic control algorithm. The computational approach, on the contrary, does not allow obtaining the results of the same degree of generality as the analytical approach, but has a larger area of modelling objects. The capacity to develop a computational model is primarily constrained by the availability of computational resources necessary to reproduce the intricate details of the modelling object. For this reason, it is a "rescue" tool for modelling the complex transportation systems, including modelling the pedestrian delays at signalized intersections [8–10]. In view of the above-mentioned, it can be concluded that the analytical model is the best model option, although its development is not always feasible. From this perspective, we can categorize the (operational-level) control algorithms with respect to their complexity of analysis (and, consequently, the feasibility of creating analytical models) as follows:

– *fixed cycle signal diagram control* (*FCD control*); such control implies signal timing according to a pre-defined time diagram comprising cyclically repeating sequences of signal activations in one and the same time period (cycle length) (see Fig. 1);

– *fixed cycle signal diagram operative correcting control (FCDoC control)*; this control is based on *FCD* control and involves the correction of the cycle signal diagram (by modifying the duration of signals and/or by skipping the activation of the signals altogether) in response to the actual traffic situation being observed with sensors; two further sub-classes can be distinguished here: *cycle length preserving control* (in which the correction preserves the cycle length) and *cycle non-preserving control*; a common representative of this class are variants of so-called actuated control;

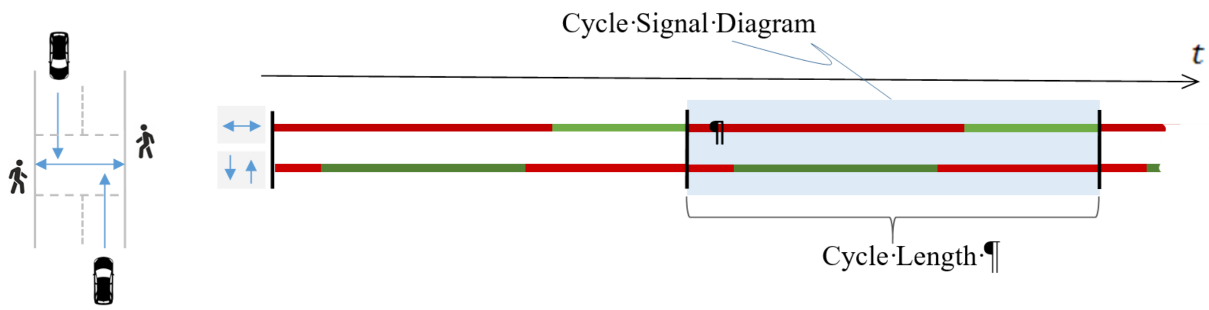

Fig. 1. Fixed cycle signal diagram control concepts

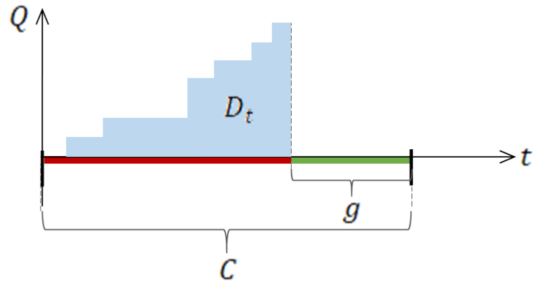

Fig. 2. Pedestrian (random) cyclic delay calculation as the area under the queue size dependence on time

– *general signal diagram operative designing control (GDoD control)*. Such control implies the building of a rather arbitrary signal diagram right during the operation process on the basis of the actual traffic situation data.

(Details of the different control variants can be found, for example, in [11, 12].)

For FCD control, it's relatively easy to derive an analytical formula for the pedestrian delay. Indeed, if we assume that the random variable $\alpha_{s:t}$ of the number of pedestrians arriving during the time interval $[s, t)$ is independent on the number of pedestrians arriving during other non-overlapping time intervals, and if we assume that the arrival process has a constant average rate q, i.e. that for the mathematically expected number of pedestrians $\mathbf{E}\alpha_{s:t}$ the relation $\mathbf{E}\alpha_{s:t} / (t-s) = q$ holds, then we can proceed as follows. It is established (see, for example, [1, 13, 14]) that the delay D_t of road-users over the time t due to waiting is numerically equal to the area under the queue size curve Q_t of the waiting users. Consequently, for the case presented in Fig. 2, the cyclic delay ${}^{FCD}MD_C$ can be calculated as follows:

$$ {}^{FCD}MD_C = \mathbf{E}\int_0^C Q_t dt = \mathbf{E}\int_0^{C-g} \alpha_{0:t} dt = \int_0^{C-g} \mathbf{E}\alpha_{0:t} dt = \int_0^{C-g} qt\, dt = \frac{1}{2}q(C-g)^2. $$

For FCDoC control algorithms, a similar mathematical derivation faces considerable difficulties. This is due to the necessity of additional taking into account already more intricate operational specifics of the algorithms. In this paper, we consider one such algorithm: cycle length-preserving pedestrian-actuated control (a common control strategy in cities, particularly as it permits the seamless integration of the pedestrian-actuated control with coordinated control of multiple intersections, providing uninterrupted traffic flow or a 'green wave'). The underlying logic is quite simple: up to a designated decision point, the control operates following a FCD control predefined signal diagram with a reserved for pedestrian service interval. When the decision point is reached, it is checked whether there is an unserved 'call for green/walk' (i. e. whether pedestrians have pressed the call button but have not yet been granted the green signal to move). If there is no call, this pedestrian interval is skipped and the unused time is transferred to serve the conflicting road users, see Fig. 3.

To the best of the authors' knowledge, the most rigorous approach to analytical modelling of the delays for such a control currently in use is the approach outlined in [15] with referring to the Highway Capacity Manual (HCM)[1]. Namely, the following line of reasoning is employed. One may consider two types of situations in the control cycle: A^+ – the first arriving pedestrians in the cycle have time to call green and thereby to activate the reserved service interval; A^- – the first pedestrians arrive later than the time when the reserved interval may be activated. It can be reasonably concluded that the delays for the pedestrians in the situation A^+ will be close to ${}^{FCD}MD_C$, while those in the situation A^- will be close to ${}^{0}MD_C = q(\Delta + C)^2 / 2,$ since the pedestrians will have to wait for the reserved interval of the next cycle (see Fig. 4).

[1] Highway Capacity Manual. Transportation Research Board. Washington, D. C., 2000. URL: https://sjnavarro.wordpress.com/wp-content/uploads/2008/08/highway_capacital_manual.pdf (accessed 5 August 2024).

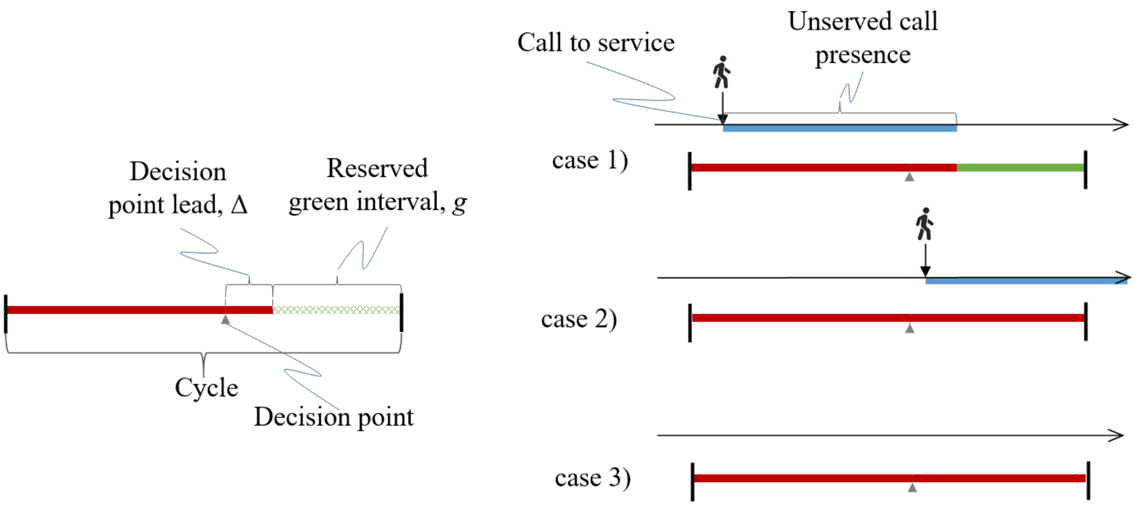

Fig. 3. Cycle length preserving pedestrian-actuated control principles

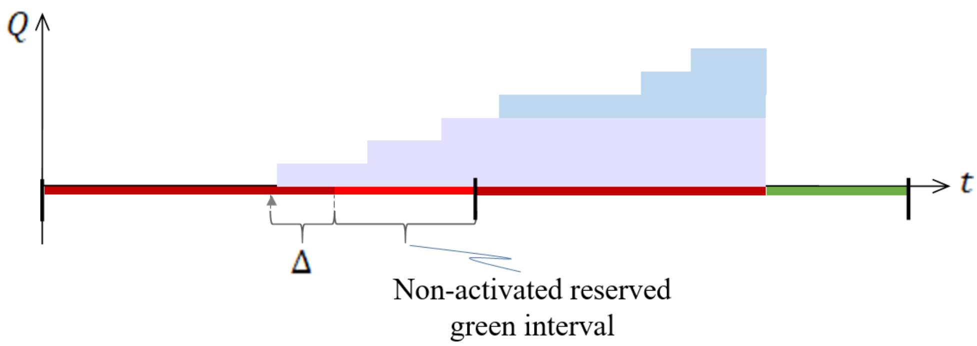

Fig. 4. Delay calculation for the situation A^{-}

Therefore, if to assume that the probability p^{+} of occurrence of the situation A^{+} is known, and that there are no other situations besides the situations A^{+} and A^{-}, then it is reasonable to find the total cyclic delay MD_C through averaging:

$$MD_C = p^{+} \cdot {}^{FCD}MD_C + (1-p^{+}) \cdot {}^{0}MD_C = {}^{FCD}MD_C + (1-p^{+}) \cdot \left({}^{0}MD_C - {}^{FCD}MD_C \right).$$

However, this approach at least fails to consider the overlap of these situations, specifically when the pedestrians in the situation A^{-}, who were unable to activate the service interval in their cycle, assist in activating it for the pedestrians in the subsequent cycle. It is therefore evident that this approach cannot be considered entirely correct.

It should also be noted that the original paper does not account for the decision point lead value. This is presumably due to the fact that in the control systems with which the authors have dealt (in particular, the North American RBC control system[1]), this lead is either small or absent. Conversely, in control systems based on stages and interstages (common in Europe [12]), this value can reach tens of seconds.

Henceforth, we will refer to the version of the HCM model in which the decision point is taken into account as the "HCM$^{\Delta}$ model".

The goal of this paper is to present a valid model that is free from such a kind of problems.

The paper is structured as follows. Initially, we develop the analytical model of pedestrian delays, then we validate it using a simulation experiment. After that, we perform a primary analysis of the re-

[1] NCHRP Report 812, Signal Timing Manual, Second Edition, Affiliation: Transportation Research Board, 2015. URL: https://transops.s3.amazonaws.com/uploaded_files/Signal%20Timing%20Manual%20812.pdf (accessed 5 August 2024).

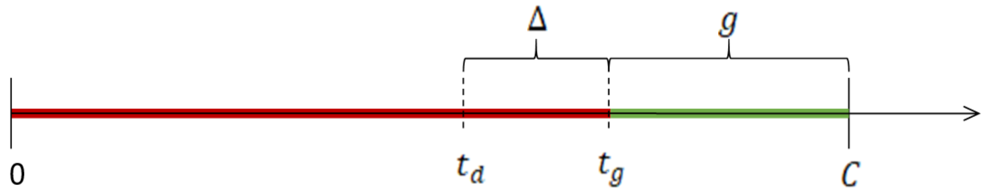

Fig. 5. The control parameters (t_d is the decision point, (t_g is the green / ”walk” start point)

sulting model (among other things, to demonstrate the advantages of analytical models over computational models). Finally, conclusions are drawn.

Model development. In this section, we employ the formal apparatus of probability theory to derive rigorous expressions for pedestrian delays under the pedestrian-actuated control. In order to develop the desired analytical model of the pedestrian delay, we first introduce the parameters of the control under consideration as it is shown in Fig. 5.

Also we assume:

A1) pedestrian arrivals are a Poisson point process with a constant rate q;

A2) pedestrian queuing processes within all cycles are statistically identical (stationary).

In addition, the following notations are to be introduced:

– $\alpha_{s:t}$ is a random variable of the pedestrian arrival Poisson point process value at a time t since a time s (i. e. arrivals number by the time t considered as if the arrivals process observation started since the time s only);

– ν is a random variable of the pedestrian queue size at the beginning of the cycle;

– ζ is a random variable of the cyclic pedestrians delay (that is mathematically equal to the area under the graph of the dependence of pedestrian queue size on time within the cycle).

We recall that the Poisson process has the following important properties, which we will make extensive use of later on in this section:

– “memoryless”:

$$\mathbf{P}(\alpha_{s:t}=k \mid \alpha_{-\infty:s}=l)=\mathbf{P}(\alpha_{s:t}=k)=\frac{\left(q(t-s)\right)^k}{k!}e^{-q(t-s)}, \quad k,l=0,1,\ldots;$$

– stationarity:

$$\mathbf{P}(\alpha_{s:t}=k)=\mathbf{P}(\alpha_{s':t'}=k), \quad k=0,1,\ldots,$$

for any non-overlapping intervals $[s,t)$, $[s',t')$, $t-s=t'-s'$.

To achieve the goal, we first need to find the probability distribution $P_\nu(k)$, $k=0,1,\ldots$, of a random variable ν. We commence by ascertaining the value $P_\nu(0)$. To do this, we introduce the following events: Z_0 = “the queue is zero at $t=0$”, and Z_C represents “the queue is zero at $t=C$”. Using the law of total probability, we can write:

$$\mathbf{P}(Z_C)=\mathbf{P}(Z_0)\mathbf{P}(Z_C \mid Z_0)+\mathbf{P}(\overline{Z}_0)\mathbf{P}(Z_C \mid \overline{Z}_0). \tag{1}$$

According to the assumption A2) we have $\mathbf{P}(Z_C)=\mathbf{P}(Z_0)=P_\nu(0)$. Therefore, with taking into account $\mathbf{P}(\overline{Z}_0)=1-\mathbf{P}(Z_0)$, we can rewrite (1):

$$P_\nu(0)=P_\nu(0)\mathbf{P}(Z_C \mid Z_0)+\left(1-P_\nu(0)\right)\mathbf{P}(Z_C \mid \overline{Z}_0). \tag{2}$$

According to the control logic, the only case when Z_C doesn’t occur (i. e. $\overline{Z}_C$ occurs), conditioned the event Z_0 has occurred, is when there are no arrivals until the decision point t_d, and there is some until the end of the cycle. Consequently, we can write:

$$\mathbf{P}(Z_C \mid Z_0)=1-\mathbf{P}(\overline{Z}_C \mid Z_0)=1-\mathbf{P}(\text{“}\alpha_{0:t_d}=0\text{” and “}\alpha_{t_d:C}\neq 0\text{”})=$$
$$=1-\mathbf{P}(\text{“}\alpha_{0:t_d}=0\text{”})\mathbf{P}(\text{“}\alpha_{t_d:C}\neq 0\text{”})=1-e^{-qt_d}\left(1-e^{-q(C-t_d)}\right).$$

Similarly, we can conclude

$$\mathbf{P}(Z_C \mid \overline{Z}_0) = 1.$$

With that said, we can obtain from (2) the expression for $P_\nu(0)$:

$$P_\nu(0) = \frac{1}{1 + e^{-qt_d}\left(1 - e^{-q(C-t_d)}\right)}. \tag{3}$$

Having obtained $P_\nu(0)$, now we can find the probabilities $P_\nu(r)$ for the events A_C^r = "queue size is r at $t = C$", $r = 1,2,\ldots$. Indeed, in the similar way, using the law of total probability, we can write:

$$P_\nu(r) = \mathbf{P}\left(A_C^r\right) = \mathbf{P}(Z_0)\mathbf{P}\left(A_C^r \mid Z_0\right) + \mathbf{P}(\overline{Z}_0)\mathbf{P}\left(A_C^r \mid \overline{Z}_0\right).$$

By the similar reasoning,

$$\mathbf{P}\left(A_C^r \mid Z_0\right) = \mathbf{P}(\text{"}\alpha_{0:t_d} = 0\text{" and "}\alpha_{t_d:C} = r\text{"}) = \mathbf{P}(\text{"}\alpha_{0:t_d} = 0\text{"})\mathbf{P}(\text{"}\alpha_{t_d:C} = \text{r"}) =$$

$$= e^{-qt_d} \frac{q^r (C - t_d)^r}{r!} e^{-q(C-t_d)},$$

$$\mathbf{P}\left(A_C^r \mid \overline{Z}_0\right) = 0.$$

And thus,

$$P_\nu(r) = P_\nu(0)\, e^{-qt_d} \frac{q^r (C - t_d)^r}{r!} e^{-q(C-t_d)}, \qquad r = 1,2,\ldots\ . \tag{4}$$

As a consequence, we can easily find the expected value of ν:

$$\mathbf{E}\nu = P_\nu(0) e^{-qt_d} q(C - t_d). \tag{5}$$

With the results obtained, we can now turn to deriving the mathematical expectation of the pedestrian cyclic delay ζ. To do this, we first introduce a random variable τ_1 which means the time of the first arrival of a pedestrian since the beginning of the cycle considered (irrespective of whether there was already a pedestrian from the previous cycle at the beginning of the cycle). Since random variables τ_1, ν are independent, their joint probability distribution can be represented with the (generalized) density $p_{\tau_1,\nu}(t,u) = p_{\tau_1}(t)\sum_{k=0}^{\infty} P_\nu(k)\delta(u-k)$, where $p_{\tau_1}(t) = qe^{-qt}$, δ is the Dirac delta function. With this in mind, using the law of total probability, we can write the decomposition:

$$\begin{aligned}\mathbf{E}\zeta \;&=\; \int_0^\infty\!\!\int_0^\infty p_{\tau_1,\nu}(t,u)\mathbf{E}(\zeta \mid \tau_1 = t, \nu = u)\,dt\,du \;=\; \int_0^\infty p_{\tau_1}(t) \sum_{k=0}^{\infty} P_\nu(k)\mathbf{E}(\zeta \mid \tau_1 = t, \nu = k)\,dt = \\ &=\; \int_0^\infty p_{\tau_1}(t) P_\nu(0)\mathbf{E}(\zeta \mid \tau_1 = t, \nu = 0)\,dt + \int_0^\infty p_{\tau_1}(t) \sum_{r=1}^{\infty} P_\nu(r)\mathbf{E}(\zeta \mid \tau_1 = t, \nu = r)\,dt.\end{aligned} \tag{6}$$

We further decompose the integration range of the first integral in the sum (6) into $[0,t_d)$, $[t_d,C)$, $[C,\infty)$ and the second integral - into $[0,t_g)$, $[t_g,\infty)$, and for each of them we consider the integrands:

for $t \in [0, t_d)$:

$$g_1(t) = \mathbf{E}(\zeta \mid \tau_1 = t, \nu = 0) = \mathbf{E}\int_t^{t_g} (1 + \alpha_{t:v})\,dv = \int_t^{t_g} (1 + \mathbf{E}\alpha_{t:v})\,dv = t_g - t + \frac{q(t_g - t)^2}{2},$$

for $t \in [t_d, C)$:

$$g_2(t) = \mathbf{E}(\zeta \mid \tau_1 = t, \nu = 0) = \mathbf{E}\int_t^{C} (1 + \alpha_{t:v})\,dv = \int_t^{C} (1 + \mathbf{E}\alpha_{t:v})\,dv = C - t + \frac{q(C - t)^2}{2},$$

for $t \in [C, \infty)$:

$$g_3(t) = \mathbf{E}(\zeta | \tau_1 = t, \nu = 0) = 0,$$

for $t \in [0, t_g)$:

$$h_1(t) = \sum_{r=1}^{\infty} P_\nu(r) \mathbf{E}(\zeta | \tau_1 = t, \nu = r) = \sum_{r=1}^{\infty} P_\nu(r) \mathbf{E}\left(\int_t^{t_g} (1 + \alpha_{t:v}) dv + r t_g \right) =$$

$$= \sum_{r=1}^{\infty} P_\nu(r) \int_t^{t_g} (1 + \mathbf{E}\alpha_{t:v}) dv + \sum_{r=1}^{\infty} P_\nu(r) r t_g = \left(1 - P_\nu(0)\right)\left(t_g - t + \frac{q(t_g - t)^2}{2} \right) + \mathbf{E}\nu t_g,$$

for $t \in [t_g, \infty)$:

$$h_2(t) = \sum_{r=1}^{\infty} P_\nu(r) \mathbf{E}(\zeta | \tau_1 = t, \nu = r) = \sum_{r=1}^{\infty} P_\nu(r) r t_g = \mathbf{E}\nu t_g.$$

With that in mind, we can proceed with writing (6) in the following way:

$$\mathbf{E}\zeta = P_\nu(0) \int_0^{t_d} p_{\tau_1}(t) g_1(t) dt + P_\nu(0) \int_{t_d}^{C} p_{\tau_1}(t) g_2(t) dt + 0 + \int_0^{t_g} p_{\tau_1}(t) h_1(t) dt + \int_{t_g}^{\infty} p_{\tau_1}(t) h_2(t) dt.$$

Hence, after the integration and simplification, we get *the final models (for the pedestrians cyclic delay):*

$$MD_C = \mathbf{E}\zeta = \frac{1}{2} q (C - g)^2 + \frac{1}{2} q P_0 e^{-q(C - g - \Delta)} \left(2C(g + \Delta) - g^2 \right), \tag{7}$$

$$Md_C = \frac{\mathbf{E}\zeta}{qC} = \frac{1}{2} \frac{(C - g)^2}{C} + \frac{1}{2} P_0 e^{-q(C - g - \Delta)} \left(2(g + \Delta) - \frac{g^2}{C} \right), \tag{8}$$

where

$$P_0 = \frac{1}{1 + e^{-q(C - g - \Delta)} (1 - e^{-q(g + \Delta)})} \tag{9}$$

is the probability of the event "there are no unserved calls at the end of the cycle".

Model validation with simulation. To validate the developed model, a simulation model was created to reproduce the dynamics of the pedestrian queuing and servicing process over the time $T = nC$, where n is the number of observation cycles, C is the cycle length. The variable under study was the sample average cyclic pedestrian delay, namely, $\widehat{MD}_C = D_T / T$, where D_T is the sample total pedestrian delay over the time period T. It was assumed that the random process of cyclic delays was sufficiently 'good' that the value of $\widehat{MD}_C$ could be considered as a consistent estimator of the desired cyclic delay $MD_C = \mathbf{E}\zeta$.

To design the experiment plan, a list of parameter set variants was created, including all possible combinations of values from the ranges presented in the Table below.

Parameters ranges

Parameter	Range		
	min	max	step
C, (s)	60	140	10
g, (s)	5	30	2
Δ, (s)	5	20	2
q, ($person / s$)	0.003	0.03	0.001

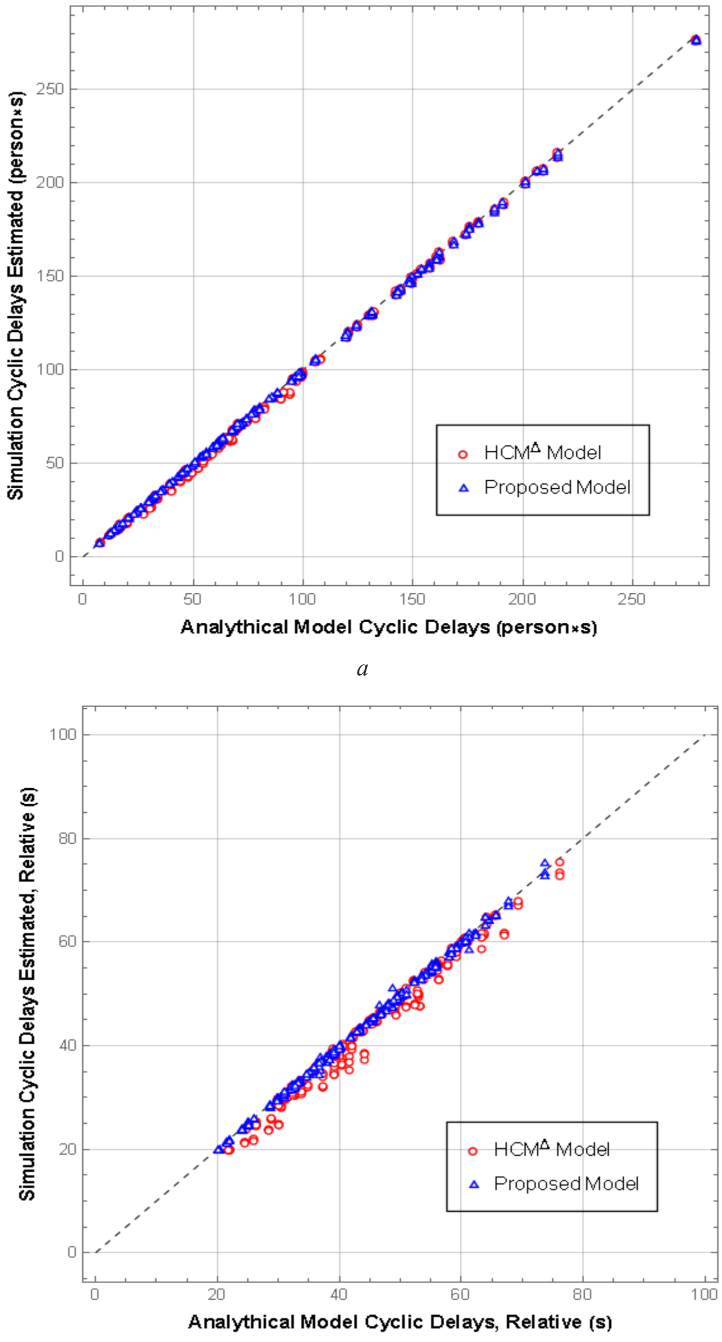

Fig. 6. Analytical model validation graphs "Predicted vs. Simulated":
a is for cyclic delays; *b* is for relative cyclic delays

Subsequently, K sets were randomly selected, with a replication of the size m being made in the experiment plan for each of them.

The values employed in the experiment were as follows: $n = 1000$, $K = 100$, $m = 3$.

The results of the experiment are presented in a graphical form in Fig. 6, *a* and 5, *b*. Fig. 6, *a* is related to total cyclic delays MD_C, while Fig. 6, *b* – to relative cyclic delays $Md_C = MD_C / (qC)$. These figures illustrate the comparison between the values predicted by the analytical models and those obtained from the simulation.

The visual representation indicates that the proposed model is in accordance with the experimental data. The results of the simple regression analysis indicate the same – the value of the free term estimate is statistically insignificant (can be put equal to zero). At the same time, in comparison, the HCM$^{\Delta}$ model gives statistically significant biased results (overestimated). The latter can be explained as follows. Since for the HCM$^{\Delta}$ model the corresponding probability p^{+} has the expression $p^{+} = 1 - e^{-q(C-g-\Delta)}$, we can rewrite this model in the following way:

$$MD_C^{\mathrm{HCM}^{\Delta}} = \frac{1}{2}q(C-g)^2 + e^{-q(C-g-\Delta)}\left(\frac{1}{2}q(\Delta+C)^2 - \frac{1}{2}q(C-g)^2\right) =$$
$$= \frac{1}{2}q(C-g)^2 + \frac{1}{2}q\,e^{-q(C-g-\Delta)}\left(2C(g+\Delta)-g^2\right) + \frac{1}{2}q\,e^{-q(C-g-\Delta)}\Delta^2. \tag{10}$$

A comparison of the expression (10) with the expression (7) reveals a discrepancy due to the presence of the multiplier P_0 in the second summand of (10) and the third summand of (10). Given that P_0 does not exceed unity, it can be concluded that $MD_C^{\mathrm{HCM}^{\Delta}}$ provides values that are higher than those of the proposed model.

It is important to note that the original, uncorrected HCM model (as it was identified during an additional experiment) yields, as anticipated, substantially biased and underestimated results.

Primary analysis. We analyze an expression for the relative delay Md_C first (since it largely reflects the average waiting time per pedestrian in one cycle). As it can be observed, the expression comprises two summands. The first one is essentially the relative delay for FCD control, while the second one represents the additional delay attributed to the implementation of the pedestrian-actuated control. We denote this additional delay by ${}^{P-A}\Delta Md_C$ and analyze it. For this purpose we introduce the following variables: $\lambda = g/C$ is a fraction of green in one cycle, $\mu = \Delta/g$ is a relative lead time, $\kappa = 1/(qC)$ is an average number of cycles between consecutive pedestrian arrivals, and rewrite the expression for ${}^{P-A}\Delta Md_C$ as:

$${}^{P-A}\Delta Md_C = \eta(\lambda,\mu,\kappa)\cdot C, \qquad \eta(\lambda,\mu,\kappa) = \frac{1}{2}\frac{e^{-\frac{1-\lambda(1+\mu)}{\kappa}}}{1+e^{-\frac{1-\lambda(1+\mu)}{\kappa}\left(1-e^{-\frac{\lambda(1+\mu)}{\kappa}}\right)}}\big(2(1+\mu)-\lambda\big)\lambda.$$

The value η can be interpreted as a fraction of the cycle time, describing the extent to which the delay is increased when the pedestrian-actuated control is implemented. To investigate this value in greater detail, we will plot the graphs (see Fig. 7).

The graphs show, for example, that for a fraction $\lambda < 0.1$, the value of η for a varying κ (and also for a varying μ) remains relatively low, not exceeding 0.05–0.1 (5–10 %) of the cycle length. Therefore, referring to a typical large cycle with an approximate length $C \approx 100$ s, we can conclude that for such λ the implementation of pedestrian-actuated control will not be a major issue, since

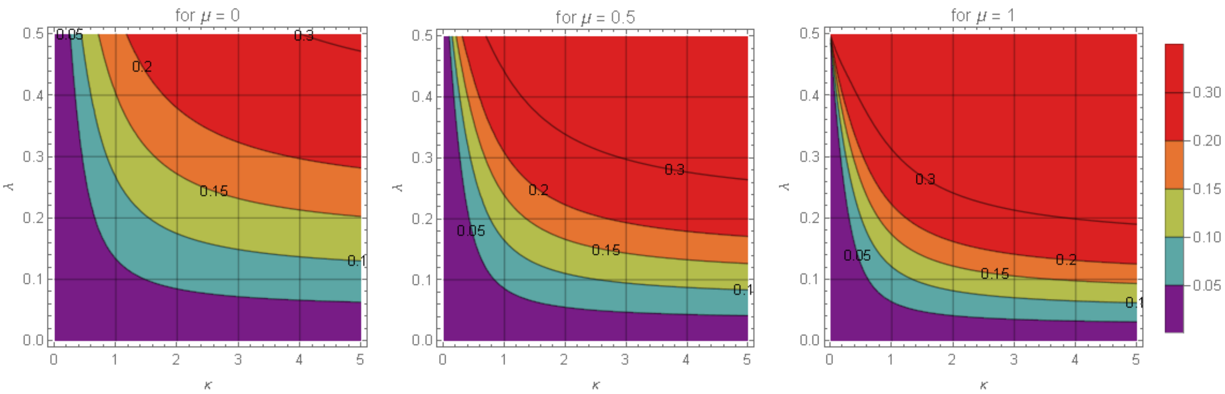

Fig. 7. Contour plots for $\eta = \eta(\lambda,\mu,\kappa)$

the additional waiting time will not exceed ≈5–10 s. Conversely, for $\lambda > 0.1$, the implementation of this control requires a detailed study of the situation (accounting for the ranges of κ, the value of μ, and the cycle length C itself).

Conclusions. The findings of the study demonstrate that the proposed analytical model exhibits greater accuracy compared to the HCM^{Δ} model (and even more so when compared to the original HCM one), while maintaining a relatively simple structure. This enables the model to be employed for the efficient analysis, particularly in the evaluation of the suitability of implementing the corresponding pedestrian-actuated control as a part of the development of intelligent transportation systems.

It is also noteworthy that the approach proposed in the paper allows, in principle, the calculation of other statistical characteristics of delays (such as variance, correlation coefficient, etc.), which may be useful in other applications.

Possible future directions of the study include: improving the model for the case when the flow of pedestrian arrivals is not a simple Poisson process, but, for example, a Compound Poisson process that takes into account clustering of pedestrians (which in urban conditions can be caused by the presence of signal controlled crossings, the presence of public transport stops near the pedestrian crossing, etc.), or inhomogeneous in time, when the flow intensity varies with time (which corresponds to the daily variation of the pedestrian traffic intensity). It is also possible to consider the application of the proposed approach to calculate pedestrian delays for the case when the control cycle length is not preserved, and other variations of the pedestrian-actuated control.

References

1. United Nations Economic Commission for Europe. *A Handbook on Sustainable Urban Mobility and Spatial Planning: Promoting Active Mobility*. UN, 2020. 234 p. https://doi.org/10.18356/8d742f54-en

2. Inose H., Hamada T. *Road Traffic Control*. University of Tokyo Press, 1975. 331 p.

3. National Academies of Sciences, Engineering, and Medicine. *Traffic Signal Control Strategies for Pedestrians and Bicyclists.* Washington, DC: The National Academies Press, 2022. https://doi.org/10.17226/26491

4. Buslenko N. P. *Modeling of Complex Systems.* Moscow, Nauka Publ., 1978. 400 p. (in Russian).

5. Sovetov B. Ya., Yakovlev S. A. *System Modeling*. Moscow, Vysshaya shkola Publ., 2007. 343 p. (in Russian).

6. Kharin Yu. S., Malyugin V. I., Kirlitsa V. P., Lobach V. I. *Fundamentals of Simulation and Statistical Modeling*. Minsk, Dizain PRO Publ., 1997. 288 p. (in Russian).

7. Akçelik R., Besley M. Microsimulation and analytical methods for modelling urban traffic. *Paper presented at the Conference on Advance Modeling Techniques and Quality of Service in Highway Capacity Analysis, Truckee, California, USA, July 2001.* 19 p.

8. Gavric S., Sarazhinsky D., Stevanovic A., Dobrota N. Development and evaluation of non-traditional pedestrian timing treatments for coordinated signalized intersections. *Transportation Research Record: Journal of the Transportation Research Board*, vol. 2677, iss. 1, pp. 460–474. https://doi.org/10.1177/03611981221099913

9. Feng-Bor Lin. A simulation analysis of pedestrian actuated traffic signal control system. *Transportation Research*, 1978, vol. 12, iss. 1, pp. 21–28. https://doi.org/10.1016/0041-1647(78)90103-x

10. Gavric S., Erdagi I. G., Stevanovic A. Environmental Assessment of Incorrect Automated Pedestrian Detection and Common Pedestrian Timing Treatments at Signalized Intersections. *Sustainability*, 2024, vol. 16, no. 11, art. ID 4487. https://doi.org/10.3390/su16114487

11. *COLOMBO Deliverable 2.2: Policy Definition and dynamic Policy Selection Algorithms, 2014.* Available at: https://elib.dlr.de/96128/1/COLOMBO_D2.2_PolicySelectionDefinition_v1.9.pdf (accessed 5 August 2024).

12. *Traffic Signs Manual: Chapter 9. Traffic Signals*. Spain, Department Transport, Tourism and Sport, 2019. Available at: https://assets.gov.ie/34735/fb85c77684d045b495b70335d8d3cf20.pdf (accessed 5 August 2024).

13. Rouphail N., Tarko A., Li J. *Traffic flow at signalized intersections, 1997.* Available at: https://www.fhwa.dot.gov/publications/research/operations/tft/chap9.pdf (accessed 5 August 2024).

14. Daley D. J., Vere-Jones D. *An Introduction to the Theory of Point Processes. Vol. 2. General Theory and Structure*. Springer, New York, 2008. https://doi.org/10.1007/978-0-387-49835-5

15. Wang X., Tian Z. Z., Ohene F., Koonce P. J. V. Pedestrian Delay Models at Signalized Intersections Considering Signal Phasing and Pedestrian Treatment Alternatives. *Presented at 88th Annual Meeting of the Transportation Research Board*. Washington, D. C., 2009. Available at: https://trid.trb.org/view.aspx?id=881786

Information about the authors

Denis S. Sarazhinsky – Associate Professor, Ph. D. (Physics and Mathematics), Associate Professor in the Department of Transport Systems and Technologies, Belarusian National Technical University (65, Nezavisimosti Ave., 220013 Minsk, Belarus). E-mail: sarazhinsky@mail.ru

Denis V. Kapski – Dr. Sc. (Engineering), Professor, Professor in the Department of Transport Systems and Technologies, Belarusian National Technical University (65, Nezavisimosti Ave., 220013, Minsk, Republic of Belarus); Professor in the Department of Information Resource Management, Academy of Public Administration under the President of the Republic of Belarus (17, Moskovskaya Str., 220007, Minsk, Republic of Belarus). E-mail: d.kapsky@bntu.by, d.kapsky@gmail.com. https://orcid.org/0000-0001-9300-3857

Информация об авторах

Саражинский Денис Сергеевич – доцент, кандидат физико-математических наук, доцент кафедры «Транспортные системы и технологии», Белорусский национальный технический университет (пр. Независимости, 65, 220013, Минск, Республика Беларусь). E-mail: sarazhinsky@mail.ru

Капский Денис Васильевич – доктор технических наук, профессор, профессор кафедры «Транспортные системы и технологии», Белорусский национальный технический университет (пр. Независимости, 65, 220013, Минск, Республика Беларусь); профессор кафедры «Управление информационными ресурсами», Академия управления при Президенте Республики Беларусь (ул. Московская, 17, 220007, Минск, Республика Беларусь). E-mail: d.kapsky@bntu.by, d.kapsky@gmail.com. https://orcid.org/0000-0001-9300-3857