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Valentina V. Skrundz, Inna N. Safonova

Belarusian State University, Minsk, Republic of Belarus

ON THE EXISTENCE OF $\mathfrak{H}_\sigma^\tau$-CRITICAL FORMATIONS OF FINITE GROUPS

Abstract. The τ-closed σ-local formations of finite groups are studied, where σ is some partition of the set of all primes, τ is an arbitrary subgroup functor. The question of the existence of $\mathfrak{H}_\sigma^\tau$-critical formations is investigated, i. e., such τ-closed σ-local formations $\mathfrak{L} \not\subseteq \mathfrak{H}$ for which all proper τ-closed σ-local subformations are contained in the class of groups $\mathfrak{H}$. It is proved that if $\mathfrak{H}$ is a σ-local formation of classical type, then every τ-closed σ-local formation $\mathfrak{F} \not\subseteq \mathfrak{H}$ has at least one $\mathfrak{H}_\sigma^\tau$-critical subformation.

Keyword: finite group, subgroup functor, τ-closed formation, formation σ-function, σ-local formation, critical σ-local formation, formation of classical type

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В. В. Скрундь, И. Н. Сафонова

Белорусский государственный университет, Минск, Республика Беларусь

О СУЩЕСТВОВАНИИ $\mathfrak{H}_\sigma^\tau$-КРИТИЧЕСКИХ ФОРМАЦИЙ КОНЕЧНЫХ ГРУПП

Аннотация. Изучены τ-замкнутые σ-локальные формации конечных групп, где σ – некоторое разбиение множества всех простых чисел, τ – произвольный подгрупповой функтор. Исследован вопрос существования $\mathfrak{H}_\sigma^\tau$-критических формаций, т. е. таких τ-замкнутых σ-локальных формаций $\mathfrak{L} \not\subseteq \mathfrak{H}$, у которых все собственные τ-замкнутые σ-локальные подформации содержатся в классе групп $\mathfrak{H}$. Доказано, что если $\mathfrak{H}$ – σ-локальная формация классического типа, то у всякой τ-замкнутой σ-локальной формации $\mathfrak{F} \not\subseteq \mathfrak{H}$ существует по меньшей мере одна $\mathfrak{H}_\sigma^\tau$-критическая подформация.

Ключевые слова: конечная группа, подгрупповой функтор, τ-замкнутая формация, формационная σ-функция, σ-локальная формация, критическая σ-локальная формация, формация классического типа

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Introduction. The study and classification of formations are inextricably linked with the investigation of questions concerning the existence of subformations of a particular kind in the studied formation and their mutual arrangement in the structure of the subformations of the given formation. Since the lattice of subformations of any non-identity local formation has the cardinality of the continuum, the application of inductive reasoning in studying the structure of subformations of a local formation is rather difficult. This circumstance led to the necessity of developing special methods for investigating local formations, associated with the concept of a critical formation. The special role of minimal local non-$\mathfrak{H}$-formations [1] or, equivalently, $\mathfrak{H}_l$-critical formations [2] (i. e., such local formations

$\mathfrak{F} \not\subseteq \mathfrak{H}$ all of whose proper local subformations are contained in the class of groups $\mathfrak{H}$) was first noted by L. A. Shemetkov at the VI All-Union Symposium on Group Theory [1]. There, he also set the general problem of studying critical formations. The solution to this problem in the class of local formations was obtained by A. N. Skiba in a series of works from 1980–1993. The main result of this series of works was the construction of a theory of minimal local non-$\mathfrak{H}$-formations, central to which are the theorem describing $\mathfrak{H}_l$-critical formations for the case when $\mathfrak{H}$ is an arbitrary formation of a classical type [3] (a formation is called a *formation of the classical type* [2] if it has a local screen such that all its non-abelian values are local), as well as the theorem on the existence of $\mathfrak{H}_l$-critical formations in every local formation $\mathfrak{F} \not\subseteq \mathfrak{H}$. The results of the theory of minimal local non-$\mathfrak{H}$-formations have been widely used in questions of classification and investigation of the structure of local formations, as well as in the study of irreducible factorizations of bounded and one-generated local formations [4].

Subsequently, during the development of the theory of τ-closed local formations [5], A. N. Skiba developed the theory of $\mathfrak{H}_l^\tau$-critical formations (a τ-closed local formation $\mathfrak{F} \not\subseteq \mathfrak{H}$ is called $\mathfrak{H}_l^\tau$*-critical* [5] or *minimal* τ*-closed local non-*$\mathfrak{H}$*-formation* if all proper τ-closed local subformations of $\mathfrak{F}$ are contained in the class of groups $\mathfrak{H}$), where τ is an arbitrary subgroup functor in the sense of the following definition.

Suppose that to each group G, a certain system of its subgroups $\tau(G)$ is assigned. Then τ is called a *subgroup functor* [5, p. 16] if $G \in \tau(G)$ and for any epimorphism $\varphi: A \to B$ and for any groups $H \in \tau(A)$ and $T \in \tau(B)$, we have $H^{\varphi} \in \tau(B)$ and $T^{\varphi^{-1}} \in \tau(A)$. A formation $\mathfrak{F}$ is called τ*-closed* if $\tau(G) \subseteq \mathfrak{F}$ for any group $G \in \mathfrak{F}$.

The development of the theory of σ-properties of groups [6, 7] and their classes, in particular, the theory of σ-local formations [8–15], has led to the necessity of studying and classifying σ-local formations, including the development of a theory of critical σ-local formations. The solution to L. A. Shemetkov's problem [1] in the class of σ-local formations is given in the works [13, 14], where a description of $\mathfrak{H}_\sigma$-critical formations is obtained for an arbitrary σ-local formation $\mathfrak{H}$ of the classical type (a formation $\mathfrak{H}$ is called a σ*-local formation of the classical type* if $\mathfrak{H}$ has a σ-local definition such that all its non-abelian values are σ-local). Note that the results on critical σ-local formations have been successfully used by the authors [16] in the study of σ-local formations with bounded $\mathfrak{H}_\sigma$-defect.

The study of minimal τ-closed σ-local non-$\mathfrak{H}$-formations, or $\mathfrak{H}_\sigma^\tau$-critical formations, is the subject of the authors' works [17, 18], where a description of minimal τ-closed σ-local non-$\mathfrak{H}$-formations is obtained for an arbitrary σ-local formation $\mathfrak{H}$ of the classical type.

T h e o r e m A [18, Theorem]. *Let* $\mathfrak{H}$ *be a* σ*-local formation of the classical type and h be its canonical* σ*-local definition. Then* $\mathfrak{F}$ *is an* $\mathfrak{H}_\sigma^\tau$*-critical formation if and only if* $\mathfrak{F} = l_\sigma^\tau \mathrm{form} G$, *where G is a monolithic* $\overline{\tau}$*-minimal non-*$\mathfrak{H}$*-group with monolith* $P = G^{\mathfrak{H}}$, *and one of the following conditions hold:*

1) *$G = P$ is a simple σ_i-group such that* $\sigma_i \notin \sigma(\mathfrak{H})$ *and* $\tau(G) \subseteq \{1, G\}$;

2) *P is a non-*σ*-primary group and G is a* $\overline{\tau}$*-minimal non-*$h(\sigma_i)$*-group with* $P = G^{h(\sigma_i)}$ *for all* $\sigma_i \in \sigma(P)$;

3) $G = P \rtimes K$, *where* $P = C_G(P)$ *is a p-group,* $p \in \sigma_i$, *and K is either a monolithic* $\overline{\tau}$*-minimal non-*$h(\sigma_i)$*-group with monolith* $Q = K^{h(\sigma_i)} \not\subseteq \Phi(K)$, *where* $\sigma_i \notin \sigma(Q)$, *or a minimal non-*$h(\sigma_i)$*-group of one of the following types:*

a) *a quaternion group of order* 8, *if* $2 \notin \sigma_i$;

b) *an extraspecial group of order* q^3 *of prime odd exponent* $q \notin \sigma_i$;

c) *a cyclic q-group,* $q \notin \sigma_i$.

In this work, in the framework of the general problem posed by L. A. Shemetkov [1], we study the question of the existence of minimal τ-closed σ-local non-$\mathfrak{H}$-formations in every τ-closed σ-local formation $\mathfrak{F} \not\subseteq \mathfrak{H}$. The main result of the work is the following theorem.

T h e o r e m B. *Let* $\mathfrak{F}$ *be a* τ*-closed* σ*-local formation, and* $\mathfrak{H}$ *be an arbitrary* σ*-local formation of the classical type. If* $\mathfrak{F} \not\subseteq \mathfrak{H}$, *then* $\mathfrak{F}$ *contains at least one minimal* τ*-closed* σ*-local non-*$\mathfrak{H}$*-subformation.*

We will prove Theorem B in Section 2 and also consider some of its most interesting corollaries.

1. Basic concepts and auxiliary results. All groups under consideration are assumed to be finite. The main definitions, notation, and a number of properties of σ-local formations are presented in the works [8–18].

Let $\sigma=\{\sigma_i \mid i\in I\}$ be some partition of the set of all primes $\mathbb{P}$, i. e., $\sigma=\{\sigma_i \mid i\in I\}$, where $\mathbb{P}=\bigcup_{i\in I}\sigma_i$ and $\sigma_i\cap\sigma_j=\varnothing$ for all $i\neq j$, G be a group, $\mathfrak{F}$ be a class of groups. Then $\sigma(G)=\{\sigma_i \mid \sigma_i\cap\pi(G)\neq\varnothing\}$; $\sigma(\mathfrak{F})=\cup_{G\in\mathfrak{F}}\sigma(G)$.

A group G is called [7]: σ-*primary* if G is a σ_i-group for some i; σ-*nilpotent* if $G=G_1\times\cdots\times G_t$ for some σ-primary groups $G_1,\ldots,G_t$; σ-*soluble* if $G = 1$ or $G\neq 1$ and every chief factor of G is σ-primary.

The classes of all σ-soluble, σ-nilpotent, σ_i-groups, and identity groups are denoted by $\mathfrak{S}_\sigma$, $\mathfrak{N}_\sigma$, $\mathfrak{G}_{\sigma_i}$, and (1), respectively.

Let $\mathfrak{F}$ be a nonempty formation. Then for any group G, $G^{\mathfrak{F}}$ denotes $\mathfrak{F}$-*residual* of G, i. e., the smallest normal subgroup of G such that the quotient group belongs to $\mathfrak{F}$.

A formation $\mathfrak{F}$ is called *(normally) hereditary* if $H\in\mathfrak{F}$ whenever $G\in\mathfrak{F}$ and H is a (normal) subgroup of G.

Let $\mathfrak{F}$ and $\mathfrak{H}$ be some classes of groups. The *product* $\mathfrak{F}\mathfrak{H}$ of the classes of groups $\mathfrak{F}$ and $\mathfrak{H}$ is defined by the condition: $G\in\mathfrak{F}\mathfrak{H}$ if and only if there exists a normal subgroup N in G such that $G/N\in\mathfrak{F}$. If, in addition, $\mathfrak{H}$ is a formation, then the *residual product* $\mathfrak{F}\circ\mathfrak{H}$ of the classes $\mathfrak{F}$ and $\mathfrak{H}$ is defined as follows: $\mathfrak{F}\circ\mathfrak{H}=(G \mid G^{\mathfrak{H}}\in\mathfrak{F})$.

The symbol $O_{\sigma_i',\sigma_i}(G)$ denotes $(\mathfrak{G}_{\sigma_i'}\mathfrak{G}_{\sigma_i})$-radical of the group G (the largest normal $(\mathfrak{G}_{\sigma_i'}\mathfrak{G}_{\sigma_i})$-subgroup of G), i. e., $O_{\sigma_i',\sigma_i}(G)=G_{\mathfrak{G}_{\sigma_i'}\mathfrak{G}_{\sigma_i}}$.

A function f of the form $f:\sigma\to\{\text{formations of groups}\}$ is called a *formation* σ-*function*. For every formation σ-function f, the class $LF_\sigma(f)$ is defined as follows:

$$LF_\sigma(f)=\left(G \mid G=1 \text{ or } G\neq 1 \text{ and } G/O_{\sigma_i',\sigma_i}(G)\in f(\sigma_i) \text{ for all } \sigma_i\in\sigma(G)\right).$$

If for some formation σ-function f we have $\mathfrak{F}=LF_\sigma(f)$, then the formation $\mathfrak{F}$ is said to be σ-*local*, and f is called a σ-*local definition* of $\mathfrak{F}$.

If f is a formation σ-function, then Supp(f) denotes the *support* of f, i. e., the set of all σ_i such that $f(\sigma_i)\neq\varnothing$.

A formation σ-function f is called: τ-*valued* if $f(\sigma_i)$ is a τ-closed formation for all $\sigma_i\in\text{Supp}(f)$; *integrated* if $f(\sigma_i)\subseteq LF_\sigma(f)$ for all i; *full* if $f(\sigma_i)=\mathfrak{G}_{\sigma_i}f(\sigma_i)$ for all i. If f is a full integrated formation σ-function and $\mathfrak{F}=LF_\sigma(f)$, then f is called the *canonical* σ-*local definition* of $\mathfrak{F}$. The *smallest* σ-*local definition* of $\mathfrak{F}$ is its σ-local definition f such that $f(\sigma_i)\subseteq h(\sigma_i)$ for any σ-local definition h of the formation $\mathfrak{F}$ and all i.

A subgroup functor τ is called [5, p. 16]: *closed* if for any two groups G and $H\in\tau(G)$ we have $\tau(H)\subseteq\tau(G)$; *trivial* if for any group G we have $\tau(G)=\{G\}$; *identity* if for any group G we have $\tau(G)$ being the set of all subgroups of G.

A partial order ≤ is introduced on the set of all subgroup functors [5, p. 19] by defining $\tau_1\leq\tau_2$ if and only if for any group G we have $\tau_1(G)\subseteq\tau_2(G)$. For any collection of subgroup functors $\{\tau_i \mid i\in I\}$, their intersection $\tau=\bigcap_{i\in I}\tau_i$ is defined by: $\tau(G)=\bigcap_{i\in I}\tau_i(G)$ for any group G.

If τ is an arbitrary subgroup functor, then $\overline{\tau}$ denotes the intersection of all closed subgroup functors τ_i for which $\tau\leq\tau_i$. The functor $\overline{\tau}$ is called the *closure of the functor* τ [5, p. 20].

Let $\mathfrak{H}$ be a class of groups. A group G is called [5, p. 38] a τ-*minimal non*-$\mathfrak{H}$- *group* if $G\notin\mathfrak{H}$, but every proper τ-subgroup of G belongs to the class $\mathfrak{H}$.

For an arbitrary set of groups $\mathfrak{X}$ and any $\sigma_i\in\sigma$, the symbol $\mathfrak{X}(\sigma_i)$ [11, p. 962] denotes the class of groups defined as follows: $\mathfrak{X}(\sigma_i)=(G/O_{\sigma_i',\sigma_i}(G) \mid G\in\mathfrak{X})$, if $\sigma_i\in\sigma(\mathfrak{X})$, and $\mathfrak{X}(\sigma_i)=\varnothing$, if $\sigma_i\notin\sigma(\mathfrak{X})$.

The set of all τ-closed σ-local formations is denoted by l_σ^τ. Formations from l_σ^τ are called l_σ^τ-*formations.* For any collection of groups $\mathfrak{M}$, $l_\sigma^\tau\,\text{form}\,\mathfrak{M}$ denotes a τ-closed σ-local formation generated by $\mathfrak{M}$, i. e., the intersection of all l_σ^τ-formations containing $\mathfrak{M}$. In the case when $\mathfrak{M}=\{G\}$, the formation $l_\sigma^\tau\,\text{form}\,G$ is called a *one-generated* l_σ^τ-*formation.*

To prove the main result of the work, we will need some known facts from the formation theory.

L e m m a 1 [11, Lemma 2.1]. *Let f and h be formation* σ-*functions and let* Π = Supp(f). *Suppose that* $\mathfrak{F}=LF_\sigma(f)=LF_\sigma(h)$. *Then:*

(1) $\Pi=\sigma(\mathfrak{F})$;

(2) $\mathfrak{F} = (\bigcap_{\sigma_i \in \Pi} \mathfrak{G}_{\sigma_i'} \mathfrak{G}_{\sigma_i} f(\sigma_i)) \cap \mathfrak{G}_{\Pi}$. *Consequently,* $\mathfrak{F}$ *is a saturated formation*;

(3) *If every group in* $\mathfrak{F}$ *is* σ*-soluble, then* $\mathfrak{F} = \left(\bigcap_{\sigma_i \in \Pi} \mathfrak{S}_{\sigma_i'} \mathfrak{G}_{\sigma_i} f(\sigma_i)\right) \cap \mathfrak{S}_{\Pi}$;

(4) *If* $\sigma_i \in \Pi$, *then* $\mathfrak{G}_{\sigma_i}\left(f(\sigma_i) \cap \mathfrak{F}\right) = \mathfrak{G}_{\sigma_i}\left(h(\sigma_i) \cap \mathfrak{F}\right) \subseteq \mathfrak{F}$;

(5) $\mathfrak{F} = LF_{\sigma}(F)$, *where F is the unique formation* σ*-function such that* $F(\sigma_i) = \mathfrak{G}_{\sigma_i} F(\sigma_i) \subseteq \mathfrak{F}$ *for all* $\sigma_i \in \Pi$ *and* $F(\sigma_i) = \varnothing$ *for all* $\sigma_i \in \Pi'$. *Moreover,* $F(\sigma_i) = \mathfrak{G}_{\sigma_i}\left(f(\sigma_i) \cap \mathfrak{F}\right)$ *for all i.*

L e m m a 2 [4, Ch. II; 20, Ch. IV; 21, 2.2.11]. *Let* $\mathfrak{F}$, $\mathfrak{H}$ *and* $\mathfrak{M}$ *be formations. Then the following statements hold:*

(1) $\mathfrak{F} \circ \mathfrak{H} \subseteq \mathfrak{F}\mathfrak{H}$, *and* $\mathfrak{H} \subseteq \mathfrak{F} \circ \mathfrak{H}$, *if* $\mathfrak{F}$ *is nonempty;*

(2) *if* $\mathfrak{F}$ *is normally hereditary, then* $\mathfrak{F} \circ \mathfrak{H} = \mathfrak{F}\mathfrak{H}$, *and* $\mathfrak{F} \subseteq \mathfrak{F} \circ \mathfrak{H}$, *if* $\mathfrak{H}$ *is nonempty;*

(3) $\mathfrak{F} \circ \mathfrak{H}$ *is a formation;*

(4) $G^{\mathfrak{F} \circ \mathfrak{H}} = (G^{\mathfrak{H}})^{\mathfrak{F}}$ *for all* $G \in \mathfrak{G}$;

(5) $(\mathfrak{F} \circ \mathfrak{H}) \circ \mathfrak{M} = \mathfrak{F} \circ (\mathfrak{H} \circ \mathfrak{M})$.

L e m m a 3 [10, Lemma 2.4]. *If* $\mathfrak{F} = LF_{\sigma}(f)$, *then* $\mathfrak{F} = LF_{\sigma}(t)$, *where* $t(\sigma_i) = f(\sigma_i) \cap \mathfrak{F}$ *for all* $\sigma_i \in \sigma$.

L e m m a 4 [11, Theorem 1.13]. *The set* $\mathcal{S}_n^{\sigma}$ *of all n-multiply* σ*-local formations forms a subsemigroup of the semigroup of all formations* $G\mathfrak{G}$. *Moreover,* $|\mathcal{S}_n^{\sigma}| = 2^{\aleph_0}$ *for each* σ *with* $|\sigma| > 1$ *and* $\mathfrak{G}_{\sigma_i}$ *is a minimal idempotent in* $\mathcal{S}_n^{\sigma}$ *for all* $n > 0$ *and* $i \in I$.

L e m m a 5 [4, Lemma 18.8]. *If a group G has only one minimal normal subgroup and* $O_p(G) = 1$ *(p is a prime number), then there exists a faithful irreducible* F_pG*-module, where* F_p *is the field of p elements.*

L e m m a 6 [12, Lemma 11]. *If* $\mathfrak{H} = LF_{\sigma}(f)$ *and* $G / O_{\sigma_i}(G) \in f(\sigma_i) \cap \mathfrak{H}$ *for some* $\sigma_i \in \sigma(G)$, *then* $G \in \mathfrak{H}$.

Recall that the group G is called π-closed if it has a normal Hall π-subgroup.

L e m m a 7 [19, Lemma 4.4]. *Let* $K \subseteq N \trianglelefteq G$, $K \trianglelefteq G$, $K \subseteq \Phi(G)$. *Then the following statements hold:*

1) *if N/K is* π*-closed, then N is* π*-closed;*

2) $F_p(N / K) = F_p(N) / K$.

L e m m a 8 [3, Lemma 8] *Every non-abelian* τ*-closed formation contains at least one minimal* τ*-closed non-abelian subformation.*

Recall that $\tau\mathrm{form}\,\mathfrak{X}$ denotes the τ-closed formation generated by the class of groups $\mathfrak{X}$, i. e., the intersection of all τ-closed formations containing $\mathfrak{X}$.

L e m m a 9 [5, Lemma 2.2.4]. *A formation* $\mathfrak{F}$ *is a minimal* τ*-closed non-abelian formation if and only if* $\mathfrak{F} = \tau\mathrm{form}A$, *where A is a* τ*-minimal non-abelian group and one of the following conditions hold:*

1) *A is a monolithic group with a non-Frattini monolith R = A';*

2) *A is a quaternion group of order 8;*

3) *A is a nilpotent group of class 2 and its exponent is 4;*

4) *A is an extraspecial p-group of order* p^3 *and its exponent is* $p > 2$;

5) *A is a nilpotent group of class 2 with a prime odd exponent.*

L e m m a 10 [4, Corollary 19.14]. *A group is nilpotent if and only if all subformations of the formation it generates are hereditary.*

2. Main result. Recall the concept of a direct decomposition of a formation (see [5, p. 171]). Let $\{\mathfrak{F}_j \mid j \in J\}$ be a non-empty set of subclasses $\mathfrak{F}_j \subseteq \mathfrak{F}$ such that $\mathfrak{F}_{j_1} \cap \mathfrak{F}_{j_2} = (1)$ for any $j_1 \neq j_2$ from J. If, moreover, every group $G \in \mathfrak{F}$ has the form $G = A_{j_1} \times \ldots \times A_{j_t}$, where $A_{j_1} \in \mathfrak{H}_{j_1}, \ldots, A_{j_t} \in \mathfrak{F}_{j_t}$ for some $j_1, \ldots, j_t \in J$, then we write $\mathfrak{F} = \oplus_{j \in J} \mathfrak{F}_j$ (in particular, $\mathfrak{F} = \mathfrak{F}_1 \oplus \ldots \oplus \mathfrak{F}_t$, if $J = \{1, \ldots, t\}$).

L e m m a 11. *Let* $\mathfrak{F} = LF_{\sigma}(f)$, $\Pi = \sigma(\mathfrak{F})$. *Then* $\mathfrak{F} \cap \mathfrak{N}_{\sigma} = \mathfrak{N}_{\Pi}$ *and* $\Pi = \{\sigma_i \mid f(\sigma_i) \neq \varnothing\}$.

P r o o f. According to Lemma 1(1), we have $\Pi = \{\sigma_i \mid f(\sigma_i) \neq \varnothing\}$. Since the formation $\mathfrak{G}_{\sigma_i}$ is hereditary, it follows that $\mathfrak{G}_{\sigma_i} \subseteq \mathfrak{G}_{\sigma_i}(f(\sigma_i) \cap \mathfrak{F})$ by Lemma 2. Therefore, by Lemma 1(5), we have

$$\mathfrak{G}_{\sigma_i} \subseteq \mathfrak{G}_{\sigma_i}(f(\sigma_i) \cap \mathfrak{F}) \subseteq \mathfrak{F}$$

for every $\sigma_i \in \Pi$ Hence, $\mathfrak{N}_{\Pi} = \oplus_{\sigma_i \in \Pi} \mathfrak{G}_i \subseteq \mathfrak{F}$.

On the other hand, it is obvious that $\mathfrak{F} \cap \mathfrak{N}_{\sigma} \subseteq \mathfrak{N}_{\Pi}$. Therefore, $\mathfrak{F} \cap \mathfrak{N}_{\sigma} = \mathfrak{N}_{\Pi}$. The lemma is proved.

L e m m a 12. *Let h be the canonical definition of* $\mathfrak{H}$ *and* $G = P \rtimes K$ *be a monolithic group with the monolith* $P = C_G(P) = G^{\mathfrak{H}}$ *being a p-group,* $p \in \sigma_i \in \sigma$, $O_{\sigma_i}(K) = 1$. *Then G is a* $\overline{\tau}$*-minimal non-*$\mathfrak{H}$*-group if and only if K is a* $\overline{\tau}$*-minimal non-*$h(\sigma_i)$*-group.*

P r o o f. N e c e s s i t y. Let G be a $\overline{\tau}$-minimal non-$\mathfrak{H}$-group and $M \in \overline{\tau}(K) \setminus \{K\}$. If under the isomorphism $G / P \simeq K$ the subgroup M corresponds to the subgroup D/P in G/P, then $D \in \overline{\tau}(G) \setminus \{G\}$. Consequently, $D \in \mathfrak{H}$. Therefore, $D / O_{\sigma_{i'},\sigma_i}(D) \in h(\sigma_i)$. Then since

$$D = D \cap G = D \cap PK = P \rtimes (D \cap K), \quad P = C_G(P) \text{ and } O_{\sigma_i}(K) = 1,$$

it follows that $O_{\sigma_{i'},\sigma_i}(D) = O_{\sigma_i}(D)$. It is also clear that $O_{\sigma_i}(D) / P = O_{\sigma_i}(D / P)$. Hence, since $D / O_{\sigma_{i'},\sigma_i}(D) \in h(\sigma_i)$, we have $M / O_{\sigma_i}(M) \in h(\sigma_i) = \mathfrak{G}_{\sigma_i} h(\sigma_i)$. Therefore, $M \in h(\sigma_i)$. Thus, every proper $\overline{\tau}$-subgroup of K belongs to $h(\sigma_i)$. If, moreover, the group $K \in h(\sigma_i)$, then $G \in \mathfrak{H}$ by Lemma 6, which is impossible. Hence, K is a $\overline{\tau}$-minimal non-$h(\sigma_i)$-group.

S u f f i c i e n c y. Let K be a $\overline{\tau}$-minimal non-$h(\sigma_i)$-group and $M \in \overline{\tau}(G) \setminus \{G\}$. Let us show that $M \in \mathfrak{H}$. If $G = PM$, then obviously $M \cap P = 1$. Hence, $M \simeq M / P \cap M \simeq MP / P = G / P \in \mathfrak{H}$. Suppose $PM \neq G$. Then $PM / P \in \overline{\tau}(G / P) \setminus \{G / P\}$. But $G / P \simeq K$. Consequently, there exists a proper $\overline{\tau}$-subgroup D in K such that $MP / P \simeq D \in h(\sigma_i)$. Thus,

$$M / M \cap P \simeq MP / P \in h(\sigma_i) = \mathfrak{G}_{\sigma_i} h(\sigma_i).$$

Therefore, $M \in h(\sigma_i)$. Hence, every proper $\overline{\tau}$-subgroup of G belongs to $\mathfrak{H}$. Since by condition $P = G^{\mathfrak{H}}$, it follows that G is a $\overline{\tau}$-minimal non-$\mathfrak{H}$-group. The lemma is proved.

P r o o f of Theorem B. Assume first that $\sigma(\mathfrak{F}) \not\subseteq \sigma(\mathfrak{H})$ and let $\sigma_i \in \sigma(\mathfrak{F}) \setminus \sigma(\mathfrak{H})$. Then by Lemma 1(1)(4), we have $\mathfrak{G}_{\sigma_i} \not\subseteq \mathfrak{H}$ and $\mathfrak{G}_{\sigma_i} \subseteq \mathfrak{F}$. Since, moreover, $\mathfrak{G}_{\sigma_i}$ is a hereditary σ-local formation, it follows that $\mathfrak{G}_{\sigma_i}$ is a τ-closed σ-local formation for any subgroup functor τ. Since the formation (1) of all identity groups is the only proper τ-closed σ-local subformation of the formation $\mathfrak{G}_{\sigma_i}$, and $(1) \subseteq \mathfrak{H} \neq \varnothing$, it follows that $\mathfrak{G}_{\sigma_i}$ is the desired $\mathfrak{H}^{\tau}_{\sigma}$-critical formation contained in $\mathfrak{F}$.

Henceforth, we assume that $\sigma(\mathfrak{F}) \subseteq \sigma(\mathfrak{H})$. Let f be the smallest τ-valued σ-local definition of $\mathfrak{F}$, and h be the canonical σ-local definition of $\mathfrak{H}$. Set $\mathcal{S} := \{\sigma_j \in \sigma(\mathfrak{F}) \mid f(\sigma_j) \not\subseteq h(\sigma_j)\}$. Since $\mathfrak{F} \not\subseteq \mathfrak{H}$, it follows that $\mathcal{S} \neq \varnothing$. Associate with each $\sigma_j \in \mathcal{S}$ a group H_j, which is a group of minimal order from $f(\sigma_j) \setminus h(\sigma_j)$. Let $\mathcal{H} := \{H_j \mid \sigma_j \in \mathcal{S}\}$ and $H = H_i$ be a group of the smallest order among the groups in the set $\mathcal{H}$ Denote by D the monolith of H. By condition, $\mathfrak{H}$ has a σ-local definition k such that every nonempty value of it is either an abelian or a σ-local formation. By Lemma 3, we may assume that the σ-local definition k is integrated. By Lemma 1(5), we have $h(\sigma_r) = \mathfrak{G}_{\sigma_r} k(\sigma_r)$ for any σ_r. Since $\sigma_i \in \sigma(\mathfrak{F}) \subseteq \sigma(\mathfrak{H})$, it follows that $h(\sigma_i) = \mathfrak{G}_{\sigma_i} k(\sigma_i) \neq \varnothing$ by Lemma 1(1). Therefore, $k(\sigma_i) \neq \varnothing$.

First, consider the case when $k(\sigma_i)$ is a σ-local formation. Then by Lemma 4, $h(\sigma_i)$ is also σ-local. Since by Lemma 1(2) the formation $h(\sigma_i)$ is saturated, it follows that $D \not\subseteq \Phi(H)$.

Assume that D is a non-σ-primary group. Then $O_{\sigma_{j'},\sigma_j}(H) = 1$ for all $\sigma_j \in \sigma(D)$. Suppose that $\sigma_i \notin \sigma(D)$ and let $p \in \sigma_i$. Then since H is monolithic and $O_p(H) = 1$, by Lemma 5 there exists a faithful irreducible F_pH-module K. Let $G = K \rtimes H$. Clearly, $K \subseteq O_{\sigma_i}(G)$. Since $O_{\sigma_i}(H) = 1$, we have

$$O_{\sigma_i}(G) = O_{\sigma_i}(G) \cap G = O_{\sigma_i}(G) \cap KH = K\left(O_{\sigma_i}(G) \cap H\right) = K.$$

Since, moreover, $G / O_{\sigma_i}(G) = G / K \simeq H \in f(\sigma_i) \subseteq \mathfrak{F}$, it follows that $G \in \mathfrak{F}$ by Lemma 6. Hence, $\mathfrak{D} = l^{\tau}_{\sigma}\mathrm{form}G \subseteq \mathfrak{F}$. By the choice of H, we have $D = H^{h(\sigma_i)}$ and H is a $\overline{\tau}$-minimal non-$h(\sigma_i)$-group. Therefore, G is a $\overline{\tau}$-minimal non-$\mathfrak{H}$-group by Lemma 12.

If $K = G^{\mathfrak{H}}$, then $\mathfrak{D}$ is the desired $\mathfrak{H}^{\tau}_{\sigma}$-critical formation contained in $\mathfrak{F}$ by Theorem A(3).

Suppose $G / K \notin \mathfrak{H}$. Then since $H \simeq G / K$, it follows that $H \notin \mathfrak{H}$. Consider the formation $\mathfrak{L} = l^{\tau}_{\sigma}\mathrm{form}H$. By our assumption, $H^{\mathfrak{H}} = D$. Assume that there exists $\sigma_s \in \sigma(D)$ such that $H / D \notin h(\sigma_s)$. Since $H \in \mathfrak{F}$ and $O_{\sigma_{s'},\sigma_s}(H) = 1$, we have $H \in f(\sigma_s)$. Thus, $H / D \in f(\sigma_s) \setminus h(\sigma_s)$. Since $|H / D| < |H|$, this contradicts the definition of H. Therefore, $H^{h(\sigma_s)} = D$ for every $\sigma_s \in \sigma(D)$.

Let us show that H is a $\overline{\tau}$-minimal non-$h(\sigma_s)$-group for any $\sigma_s \in \sigma(D)$. Indeed, suppose that there exist $\sigma_r \in \sigma(D)$ and a proper $\overline{\tau}$-subgroup M of H such that $M \notin h(\sigma_r)$. Then since $H \in f(\sigma_i) \subseteq \mathfrak{F}$ and $O_{\sigma_{r'},\sigma_r}(H) = 1$, we have

$$H \simeq H / O_{\sigma_{r'},\sigma_r}(H) \in f(\sigma_r) = \tau\mathrm{form}\left(A / O_{\sigma_{r'},\sigma_r}(A) \mid A \in \mathfrak{F}\right).$$

Hence, $M \in f(\sigma_r) \setminus h(\sigma_r)$ and $|M| < |H|$. The resulting contradiction shows that H is a $\overline{\tau}$-minimal non-$h(\sigma_s)$-group for every $\sigma_s \in \sigma(D)$. But then $\mathfrak{L}$ is the desired minimal τ-closed σ-local non-$\mathfrak{H}$-formation contained in $\mathfrak{F}$ by Theorem A(2).

Suppose now that $\sigma_i \in \sigma(D)$. Let $\mathfrak{D} = l_\sigma^\tau\mathrm{form}H$. Clearly, $\mathfrak{D} \subseteq \mathfrak{F}$. Since $H^{h(\sigma_i)} = D$, it follows that $H / D \in \mathfrak{H}$. If we assume that $H \in \mathfrak{H}$, then $H \simeq H / O_{\sigma_{i'},\sigma_i}(H) \in h(\sigma_i)$. This contradicts the definition of H. Therefore, $D = H^{\mathfrak{H}}$. As above, it can be shown that $H^{h(\sigma_r)} = D$ and H is a $\overline{\tau}$-minimal non-$h(\sigma_r)$-group for every $\sigma_r \in \sigma(D)$. Hence, by Theorem A(2), the formation $\mathfrak{D}$ is the desired $\mathfrak{H}_\sigma^\tau$-critical formation contained in $\mathfrak{F}$.

Suppose D is a σ-primary group, i. e., a σ_j-group for some j. Then, due to the monolithicity of H, we have $O_{\sigma_{j'},\sigma_j}(H) = O_{\sigma_j}(H)$. Since $D = H^{h(\sigma_i)}$ and $h(\sigma_i) = \mathfrak{G}_{\sigma_i}k(\sigma_i)$, it follows that $j \neq i$. Let $p \in \sigma_i$. Then by Lemma 5, there exists a faithful irreducible F_pH-module K. Let $G = K \rtimes H$. Clearly, $K \subseteq O_{\sigma_i}(G)$. Now since $O_{\sigma_i}(H) = 1$, we have

$$O_{\sigma_i}(G) = O_{\sigma_i}(G) \cap G = O_{\sigma_i}(G) \cap KH = K\left(O_{\sigma_i}(G) \cap H\right) = K.$$

Hence, since $G / K = G / O_{\sigma_i}(G) \simeq H \in f(\sigma_i)$ and f is an integrated σ-local definition of $\mathfrak{F}$, it follows that $G \in \mathfrak{F}$ by Lemma 6. Therefore, $\mathfrak{R} = l_\sigma^\tau\mathrm{form}G \subseteq \mathfrak{F}$.

Let us show that G satisfies the condition (3) of Theorem A. Assume that $G \in \mathfrak{H}$. Then

$$H \simeq G / K = G / O_{\sigma_{i'},\sigma_i}(G) \in h(\sigma_i),$$

which contradicts the definition of H. Therefore, $G \notin \mathfrak{H}$. If $H \notin \mathfrak{H}$, then by Lemma 6, we have $H / O_{\sigma_j}(H) \notin h(\sigma_j) \subseteq \mathfrak{H}$. Since $D \subseteq O_{\sigma_j}(H)$, it follows that $|H / O_{\sigma_j}(H)| < |H|$. Since the formation σ-function f is integrated, $H \in \mathfrak{H}$ and $H / O_{\sigma_{j'},\sigma_j}(H) \in f(\sigma_j)$. Hence, for some $j \neq i$, we have

$$H / O_{\sigma_{j'},\sigma_j}(H) \in f(\sigma_j) \setminus h(\sigma_j).$$

Thus, in $\mathcal{H}$ there exists a group of an order smaller than $|H|$. This contradicts the choice of H. Therefore, $H \in \mathfrak{H}$ and hence, $K = G^{\mathfrak{H}}$. Moreover, by choice, H is a $\overline{\tau}$-minimal non-$h(\sigma_i)$-group and $D = H^{h(\sigma_i)}$. Consequently, G is a $\overline{\tau}$-minimal non-$\mathfrak{H}$-group according to Lemma 12. Thus, G satisfies the condition (3) of Theorem A. Therefore, $\mathfrak{R}$ is the desired minimal τ-closed σ-local non-$\mathfrak{H}$-formation contained in $\mathfrak{F}$.

Now consider the case when $k(\sigma_i)$ is an abelian formation. If $D \not\subseteq \Phi(H)$, then by reasoning similarly to the case when $k(\sigma_i)$ is a σ-local formation, we can show that there exists a minimal τ-closed σ-local non-$\mathfrak{H}$-formation in $\mathfrak{F}$.

Suppose $D \subseteq \Phi(H)$. Since $H / D \in h(\sigma_i) = \mathfrak{G}_{\ i}k(\sigma_i)$ and H is a group of the minimal order from $f(\sigma_i) \setminus h(\sigma_i)$, it follows that $T / D := (H / D)^{k(\sigma_i)} \in \mathfrak{G}_{\sigma_i}$. Since $D \subseteq \Phi(H)$, T is σ_i-closed by Lemma 7. Hence, if $|T / D| \neq 1$, then $O_{\sigma_i}(T) \neq 1$. Since $O_{\sigma_i}(T)$ is a characteristic subgroup of T and T is normal in H, $O_{\sigma_i}(T)$ is a normal subgroup of H. But H is a monolithic group with the monolith D. Therefore, $D \subseteq O_{\sigma_i}(T)$, i. e., D is a σ_i-group. The latter is impossible, since D is a σ_j-group and $j \neq i$. Thus, $|T / D| = 1$. But then $H / D \in k(\sigma_i) \subseteq \mathfrak{N}$. Since the formation $\mathfrak{N}$ of all nilpotent groups is saturated and $D \subseteq \Phi(H)$, it follows that H is a nilpotent q-group, where $q \notin \sigma_i$.

Suppose H is a non-abelian group and $\mathfrak{D} = \mathrm{form}H$. Then $\mathfrak{D}$ is a non-abelian formation. Hence, by Lemma 8, there exists a minimal non-abelian subformation $\mathfrak{X}$ in $\mathfrak{D}$. Since $D \subseteq \Phi(H)$, by Lemma 9 we have $\mathfrak{X} = \mathrm{form}X$, where X is either a quaternion group of order 8, or an extraspecial group of order q^3 of the prime odd exponent q.

By Lemma 5, in each of these cases there exists a faithful irreducible F_pX-module R, where $p \in \sigma_i$. Let $G = R \rtimes X$. Then $O_{\sigma_{i'},\sigma_i}(G) = O_{\sigma_i}(G) = R$. Since

$$G / O_{\sigma_i}(G) = G / R \simeq X \in \mathrm{form}H \subseteq f(\sigma_i) \subseteq \mathfrak{F},$$

it follows that $G \in \mathfrak{F}$ by Lemma 6. Therefore, $\mathfrak{L} = l_\sigma^\tau\mathrm{form}G \subseteq \mathfrak{F}$. Let us show that $\mathfrak{L}$ is the desired $\mathfrak{H}_\sigma^\tau$-critical formation. In view of Theorem A(3) and Lemma 12, it suffices to prove that $R = G^{\mathfrak{H}}$ and X is a $\overline{\tau}$-minimal non-$h(\sigma_i)$-group. Let us show that $X \in \mathfrak{H}$. Since $\sigma(\mathfrak{F}) \subseteq \sigma(\mathfrak{H})$, by Lemma 11 we have $\mathfrak{N}_{\sigma(\mathfrak{F})} \subseteq \mathfrak{N}_{\sigma(\mathfrak{H})} \subseteq \mathfrak{H}$. Since X is nilpotent, and hence σ-nilpotent, it follows that $G/R \simeq X \in \mathfrak{N}_{\sigma(\mathfrak{F})} \subseteq \mathfrak{H}$. Assume that $G \in \mathfrak{H}$. Then

$$X \simeq G/R = G/O_{\sigma_{i'},\sigma_i}(G) \in h(\sigma_i) = \mathfrak{G}_{\sigma_i}k(\sigma_i).$$

Since X is a q-group and $q \notin \sigma_i$, we have $X \in k(\sigma_i)$. But in the case under consideration, the formation $k(\sigma_i)$ is abelian. The resulting contradiction shows that $G \notin \mathfrak{H}$. Therefore, $R = G^{\mathfrak{H}}$.

By Lemma 10, the formation form$H$ is hereditary. Therefore, τform$H$ = form$H$. By the choice made, H is a minimal non-$h(\sigma_i)$-group. Consequently, all proper subgroups of H belong to the abelian formation $k(\sigma_i)$.

Let X be a quaternion group of order 8. All proper subgroups of X are abelian groups of the exponent dividing 4. Since $X \in \mathrm{form}H$, the exponent of X divides the exponent of H. Hence, X is a minimal non-$h(\sigma_i)$-group. Therefore, H is a minimal non-$\mathfrak{H}$-group by Lemma 12. Thus, $\mathfrak{L}$ is the desired minimal τ-closed σ-local non-$\mathfrak{H}$-formation contained in $\mathfrak{F}$ by Theorem A(3).

Let X be an extraspecial group of order q^3 of prime odd exponent q. Then all proper subgroups of X are abelian groups of exponent q. Since $X \in \mathrm{form}H$, the exponent of X divides the exponent of H. Consequently, X is a minimal non-$h(\sigma_i)$-group. Hence, H is a minimal non-$\mathfrak{H}$-group by Lemma 12. Therefore, by Theorem A(3), $\mathfrak{L}$ is the desired $\mathfrak{H}_\sigma^\tau$-critical formation contained in $\mathfrak{F}$.

Suppose now that H is an abelian group. Then H is a cyclic q-group, where $q \in \sigma_j$. Let $p \in \sigma_i$. By Lemma 5, there exists a faithful irreducible F_pH-module R. Let $G = R \rtimes H$. Then $O_{\sigma_{i'},\sigma_i}(G) = O_{\sigma_i}(G) = R$. Since $G/O_{\sigma_i}(G) = G/R \simeq H \in f(\sigma_i) \subseteq \mathfrak{F}$, it follows that $G \in \mathfrak{F}$ by Lemma 6. Hence, $\mathfrak{M} = l_\sigma^\tau \mathrm{form}G \subseteq \mathfrak{F}$. Note that $R = G^{\mathfrak{H}}$. Indeed, if $G \in \mathfrak{H}$, then $H \simeq G/R = G/O_{\sigma_{i'},\sigma_i}(G) \in h(\sigma_i)$, which contradicts the choice of H. Therefore, $R = G^{\mathfrak{H}}$. Moreover, since, by choice, H is a minimal non-$h(\sigma_i)$-group, the group G is a minimal non-$\mathfrak{H}$-group by Lemma 12. Therefore, $\mathfrak{M}$ is the desired minimal τ-closed σ-local non-$\mathfrak{H}$-formation contained in $\mathfrak{F}$ by Theorem A(3). The theorem is proved.

In the case when τ is the identity subgroup functor, from Theorem B we obtain

C o r o l l a r y 1. *Let $\mathfrak{F}$ be a hereditary σ-local formation, $\mathfrak{H}$ be an arbitrary σ-local formation of the classical type. If $\mathfrak{F} \not\subseteq \mathfrak{H}$ then $\mathfrak{F}$ contains a minimal hereditary σ-local non-$\mathfrak{H}$-subformation.*

Let $\tau(G) = s_n(G)$ be the set of all normal subgroups of the group G for any group G. Then the following holds:

C o r o l l a r y 2. *Let $\mathfrak{F}$ be a normally hereditary σ-local formation, $\mathfrak{H}$ be an arbitrary σ-local formation of the classical type. If $\mathfrak{F} \not\subseteq \mathfrak{H}$, then $\mathfrak{F}$ contains a minimal normally hereditary σ-local non-$\mathfrak{H}$-subformation.*

In particular, if τ is the trivial subgroup functor, we have

C o r o l l a r y 3. *Let $\mathfrak{F}$ be a σ-local formation, $\mathfrak{H}$ be an arbitrary σ-local formation of the classical type. If $\mathfrak{F} \not\subseteq \mathfrak{H}$, then $\mathfrak{F}$ contains a minimal σ-local non-$\mathfrak{H}$-subformation.*

In the classical case, when $\sigma = \sigma^1 = \{\{2\},\{3\},\{5\},\ldots\}$, from Theorem B we obtain

C o r o l l a r y 4 [5, Theorem 2.3.2]. *Let $\mathfrak{F}$ be a τ-closed local formation, $\mathfrak{H}$ be an arbitrary formation of the classical type. If $\mathfrak{F} \not\subseteq \mathfrak{H}$, then $\mathfrak{F}$ contains a minimal τ-closed local non-$\mathfrak{H}$-subformation.*

If, in addition, τ is the trivial subgroup functor, we have

C o r o l l a r y 5 [3, Corollary 1]. *Let the formations $\mathfrak{F}$ and $\mathfrak{H}$ be local and $\mathfrak{F} \not\subseteq \mathfrak{H}$. If $\mathfrak{H}$ is a formation of the classical type, then $\mathfrak{F}$ contains at least one $\mathfrak{H}_l$-critical subformation.*

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References

1. Shemetkov L. A. *Screens of Step Formations. Proceedings of the VI All-Union Symposium on Group Theory.* Kiev, Naukova Dumka, 1980, pp. 37–50 (in Russian).

2. Skiba A. N. On critical formations. *Vestsi Akademii navuk BSSR. Seryya fizika-matematychnykh navuk = Proceedings of the Academy of Sciences of BSSR. Physics and Mathematics series,* 1980, vol. 4, pp. 27–33 (in Russian).

3. Skiba A. N. On critical formations. *Infinite Groups and Related Algebraic Structures*. Kiev, Institute of Mathematics of the Academy of Sciences of Ukraine, 1993, pp. 258–268 (in Russian).

4. Shemetkov L. A., Skiba A. N. *Formations of Algebraic Systems.* Moscow, Nauka Publ., 1989. 256 p. (in Russian).

5. Skiba A. N. *Algebra of Formations.* Minsk, Belaruskaya navuka Publ., 1997. 240 p. (in Russian).

6. Skiba A. N. On σ-properties of finite groups I. *Problems of Physics, Mathematics and Technics*, 2014, no. 34 (4), pp. 89–96.

7. Skiba A. N. On σ-subnormal and σ-permutable subgroups of finite groups. *Journal of Algebra,* 2015, vol. 436, pp. 1–16. https://doi.org/10.1016/j.jalgebra.2015.04.010

8. Skiba A. N. On one generalization of the local formations. *Problems of Physics, Mathematics and Technics*, 2018, no. 34 (1), pp. 79–82.

9. Chi Z., Safonov V. G., Skiba A. N. On one application of the theory of *n*-multiply σ-local formations of finite groups. *Problems of Physics, Mathematics and Technics*, 2018, no. 34 (2), pp. 85–88.

10. Zhang Ch., Skiba A. N. On Σ_t^{σ}-closed classes of finite groups. *Ukrainian Mathematical Journal,* 2019, vol. 70, no. 12, pp. 1966–1977. https://doi.org/10.1007/s11253-019-01619-6

11. Chi Z., Safonov V. G., Skiba A. N. On *n*-multiply σ-local formations of finite groups. *Communications in Algebra*, 2019, vol. 47, no. 3, pp. 957–968. https://doi.org/10.1080/00927872.2018.1498875

12. Safonova I. N., Safonov V. G. On some properties of the lattice of totally σ-local formations of finite groups *Zhurnal Belorusskogo gosudarstvennogo universiteta. Matematika. Informatika = Journal of the Belarusian State University. Mathematics and Informatics,* 2020, no. 3, pp. 1–14. https://doi.org/10.33581/2520-6508-2020-3-6-16

13. Safonova I. N. On minimal σ-local non-$\mathfrak{H}$-formations of finite groups. *Problems of Physics, Mathematics and Technics,* 2020, no. 4 (45), pp. 105–112.

14. Safonova I. N. On critical σ-local formations of finite groups. *Trudy Instituta matematiki NAN Belarusi = Proceedings of the Institute of Mathematics of the National Academy of Sciences of Belarus,* 2023, vol. 31, no. 2, pp. 63–80.

15. Safonova I. N. On *n*-multiply σ-locality of a non-empty τ-cloused formation of finite groups. *Trudy Instituta matematiki NAN Belarusi = Proceedings of the Institute of Mathematics of the National Academy of Sciences of Belarus,* 2024, vol. 32, no. 1, pp. 32–38.

16. Safonova I. N., Skrundz V. V. On σ-local formations of finite groups with bounded $\mathfrak{H}_{\sigma}$-defect. *Problems of Physics, Mathematics and Technics,* 2025, no. 1 (62), pp. 41–62.

17. Safonova I. N., Skrundz V. V. On $\mathfrak{H}_{\sigma}^{\tau}$-critical formations of finite groups. *Problems of Physics, Mathematics and Technics,* 2025, no. 3 (64), pp. 99–111.

18. Safonova I. N., Skrundz V. V. Minimal τ-closed σ-local non-$\mathfrak{H}$-formation of finite groups. *Doklady Natsional'noi akademii nauk Belarusi = Doklady of the National Academy of Sciences of Belarus,* 2025, vol. 69, no. 5, pp. 359–366. https://doi.org/10.29235/1561-8323-2025-69-5-359-366

19. Shemetkov L. A. *Formations of finite groups.* Moscow, Nauka Publ., 1978. 272 p. (in Russian).

20. Doerk K., Hawkes T. *Finite Soluble Groups.* Berlin, New York, Walter de Gruyter, 1992. 891 p. https://doi.org/10.1515/9783110870138

21. Ballester-Bolinches A. L., Ezquerro M. *Classes of Finite Groups. Mathematics and Its Applications.* Dordrecht, Springer, 2006. 385 p. https://doi.org/10.1007/1-4020-4719-3

Information about the authors

Valentina V. Skrundz – Postgraduate Student of the Department of Higher Algebra and Information Security, Belarusian State University (Nezavisimosti Ave., 4, 220030, Minsk, Republic of Belarus). E-mail: vallik@mail.ru

Inna N. Safonova – Dr. Sc. (Physics and Mathematics), Associate Professor, Professor of the Department of Higher Algebra and Information Security, Belarusian State University (Nezavisimosti Ave., 4, 220030, Minsk, Republic of Belarus). E-mail: safonova@bsu.by. https://orcid.org/0000-0001-6896-7208

Информация об авторах

Скрундь Валентина Викторовна – аспирант кафедры высшей алгебры и защиты информации, Белорусский государственный университет (пр. Независимости, 4, 220030, Минск, Республика Беларусь). E-mail: vallik@mail.ru

Сафонова Инна Николаевна – доктор физико-математических наук, доцент, профессор кафедры высшей алгебры и защиты информации, Белорусский государственный университет, (пр. Независимости, 4, 220030, Минск, Республика Беларусь). E-mail: safonova@bsu.by. https://orcid.org/0000-0001-6896-7208