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THE NON-RELATIVISTIC PROBLEM FOR A SPIN 3/2 PARTICLE IN MAGNETIC FIELD, AND TETRAD GAUGE TRANSFORMATIONS

Abstract. In the present paper, we will focus on the non-relativistic problem for a spin 3/2 particle in magnetic field, applying cylindrical coordinates and two tetrads: Cartesian and cylindrical. Here appear six different presentations for 4-component wave functions: three Cartesian ones $L^{\text{cart}}, \Psi^{\text{cart}}, \bar{\Psi}^{\text{cart}}$ and provided by using the relevant gauge transformation $L^{\text{cyl}} = S(\phi)L^{\text{cart}}$ three different presentations in cylindrical tetrad $L^{\text{cyl}}, \Psi^{\text{cyl}}, \bar{\Psi}^{\text{cyl}}$. First, we specify the non-relativistic equation for a spin 3/2 particle in magnetic field for Cartesian tetrad in bases with non-diagonal and diagonal matrix of the third spin projection. Solutions of two types are found: the first one is associated with the operator of the orbital angular momentum; the second solution relates to the third projection of the total angular momentum. Equations arising here are solved in terms of the confluent hypergeometric functions, and the corresponding energy spectra are found. The gauge transformation is introduced which relates two tetrads: Cartesian and cylindrical; it permits us to transform the system of equations in polar coordinate from Cartesian tetrad to cylindrical one. The rules for gauge transformations of diagonalized operators of the total angular and orbital momentums are found.

Keywords: spin 3/2, non-relativitic approximation, tetrad formalism, gauge symmetry, external magnetic field, exact solutions

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НЕРЕЛЯТИВИСТСКОЕ ОПИСАНИЕ ЧАСТИЦЫ СО СПИНОМ 3/2 В МАГНИТНОМ ПОЛЕ, ТЕТРАДНЫЕ КАЛИБРОВОЧНЫЕ ПРЕОБРАЗОВАНИЯ

Аннотация. Рассмотрена задача о нерелятивистской частице со спином 3/2 в магнитном поле с использованием цилиндрических координат и двух тетрад – декартовой и цилиндрической. Здесь возникают различные представления для 4-компонентной волновой функции: три декартовых $L^{\text{cart}}, \Psi^{\text{cart}}, \bar{\Psi}^{\text{cart}}$ и получаемых с использованием калибровочного преобразования $L^{\text{cyl}} = S(\phi)L^{\text{cart}}$ три цилиндрических $L^{\text{cyl}}, \Psi^{\text{cyl}}, \bar{\Psi}^{\text{cyl}}$. Сначала вводится нерелятивистское уравнение для частицы со спином 3/2 в базисах с недиагональной и диагональной третьей проекцией спина. Найдены решения двух типов, первый соотносится с диагонализацией оператора орбитального момента, а второй – с диагонализацией оператора третьей проекции полного углового момента. Возникающие при этом дифференциальные уравнения решены в терминах вырожденных гипергеометрических функций, найдены соответствующие спектры энергии. Вводится калибровочное преобразование, которое связывает декартову и цилиндрическую тетрады и позволяет преобразовать систему уравнений в полярной координате от декартовой тетрады к цилиндрической. Определены правила калибровочных преобразований для диагонализирующихся операторов полного и орбитального углового момента.

Ключевые слова: спин 3/2, нерелятивистское приближение, тетрадный формализм, калибровочная симметрия, внешнее магнитное поле, точные решения

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Introduction. Spin 3/2 particle, after seminal papers [1–3], attracts steady attention of scientific community. For instance, see the recent papers [4–13]. In particular, a special attention was given to the non-relativistic approximation for this theory in electromagnetic and gravitational fields [11, 13].

In Minkowski space-time, one may apply the usual Cartesian coordinates, but in curved space-time models we should use curvilinear coordinates an tetrad formalism [13]. In Minkowski space, one also may apply the tetrad approach, when this a substantial role play the local Lorentz gauge transformations related to freedom in choosing tetrads. In the present paper, we will consider the problem of a spin 3/2 particle in the external magnetic field, applying cylindrical coordinates and two tetrads: Cartesian and cylindrical. In the non-relativistic problem appear six different types of 4-component waves functions [12]:

$$L^{\text{cart}} \Rightarrow \Psi^{\text{cart}} \Rightarrow \bar{\Psi}^{\text{cart}}; \quad L^{\text{cyl}} = [B(\phi) \otimes O(\phi)] L^{\text{cart}}; \quad L^{\text{cyl}} \Rightarrow \Psi^{\text{cyl}} \Rightarrow \bar{\Psi}^{\text{cyl}},$$

where $B(\phi) \otimes O(\phi)$ stands for the relevant spin-vector gauge transformation, and bar-symbol is associated with the so-called cyclic basis in which the third spin projection is given by a diagonal matrix.

Equation in Cartesian tetrad basis. The uniform magnetic field along the axis z is described by relations $A = (1/2) B \times r, F_{12} = B$; whence after recalculating to cylindrical coordinates we obtain $A_\phi = -Br^2/2$. We start with a 4-component for a spin 3/2 particle (in Cartesian tetrad basis)

$$i\hbar \frac{\partial}{\partial t} \Psi^c = -\frac{\hbar^2}{2M} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2} \left(\frac{\partial}{\partial \phi} + i \frac{e}{\hbar c} A_\phi \right)^2 + \frac{\partial^2}{\partial z^2} \right] \Psi^c - \frac{e\hbar}{2M} B S_3 \Psi^c; \tag{1}$$

in this basis, the spin matrix S_3 is not diagonal:

$$S_3 = \begin{vmatrix} -1/2 & 0 & 1 & 0 \\ 0 & +1/2 & 0 & -1 \\ 0 & 0 & +3/2 & 0 \\ 0 & 0 & 0 & -3/2 \end{vmatrix}, \quad \Psi^c = \begin{vmatrix} \Psi_1^c \\ \Psi_2^c \\ \Psi_3^c \\ \Psi_4^c \end{vmatrix}. \tag{2}$$

We can find a transformation $\bar{\Psi}^c = S \Psi^c$ to a basis in which a new matrix $\bar{S}_3$ becomes diagonal:

$$S = \begin{vmatrix} 0 & 0 & 0 & 1 \\ -2 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}, \quad S^{-1} = \begin{vmatrix} 0 & -1/2 & 0 & 1/2 \\ 1/2 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix}, \quad \bar{S}_3 = \begin{vmatrix} -3/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 3/2 \end{vmatrix}; \tag{3}$$

the new components of the wave function are given by the formulas

$$\bar{\Psi}_1^c = \Psi_4^c, \quad \bar{\Psi}_2^c = -2\Psi_1^c + \Psi_3^c, \quad \bar{\Psi}_3^c = -2\Psi_2^c + \Psi_4^c, \quad \bar{\Psi}_4^c = \Psi_3^c. \tag{4}$$

In this diagonal (or cyclic) presentation, the main equation reads as

$$i\hbar \frac{\partial}{\partial t} \bar{\Psi}^c = -\frac{\hbar^2}{2M} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2} \left(\frac{\partial}{\partial \phi} + i \frac{e}{\hbar c} A_\phi \right)^2 + \frac{\partial^2}{\partial z^2} \right] \bar{\Psi}^c - \frac{e\hbar}{2M} B \bar{S}_3 \bar{\Psi}^c, \tag{5}$$

and here we have four separated equations.

Quantum numbers *m* and *j*, Cartesian tetrad basis. One can search for solutions of two types. The first variant is associated with the operator of the orbital angular momentum $\hat{l}_3 = -i\partial_\phi$:

$$\Psi^c_m = e^{-i\epsilon t} e^{ikz} e^{im\phi} \begin{vmatrix} f_1(r) \\ f_2(r) \\ f_3(r) \\ f_4(r) \end{vmatrix}, \quad \hat{l}_3 \Psi^c_m = m \Psi^c_m, \quad m = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots . \tag{6}$$

In accordance with (5), equations for four components in cyclic basis are not linked to each other, and four independent solutions exist; all four solutions satisfy the eigenvalue equation $-i\partial_\phi \Psi^c_{(n)m} = m\Psi^c_{(n)m}$; these solutions $\Psi^c_{(n)}$ relate to different values of the third spin projection: $s_3 = -3/2, -1/2, +1/2, +3/2$.

The second variant is associated with the third projection of the total angular momentum, $\hat{J}^c_3 = -i\partial_\phi + \overline{S}_3$. First, we will consider the eigenvalue problem in cyclic basis

$$\hat{J}^c_3 \overline{\Phi}^c_j = j \overline{\Phi}^c_j, \quad \overline{\Psi}^c_j = e^{-i\epsilon t} e^{ikz} \begin{vmatrix} e^{i\sigma 1\phi} g_1 \\ e^{i\sigma 2\phi} g_2 \\ e^{i\sigma 3\phi} g_3 \\ e^{i\sigma 4\phi} g_4 \end{vmatrix} \quad (j \equiv j_3). \tag{7}$$

The function $\overline{\Psi}^c_j$ from (7) is eigenvalue one for the operator $\hat{J}^c_3$, only if

$$\begin{aligned} &\sigma_1 - 3/2 = j \Rightarrow \sigma_1 = j + 3/2, \\ &\sigma_2 - 1/2 = j \Rightarrow \sigma_2 = j + 1/2, \\ &\sigma_3 + 1/2 = j \Rightarrow \sigma_3 = j - 1/2, \\ &\sigma_4 + 3/2 = j \Rightarrow \sigma_4 = j - 3/2; \end{aligned} \quad \overline{\Psi}^c_j = e^{-i\epsilon t} e^{ikz} \begin{vmatrix} e^{i(j+3/2)\phi} g_1(r) \\ e^{i(j+1/2)\phi} g_2(r) \\ e^{i(j-1/2)\phi} g_3(r) \\ e^{i(j-3/2)\phi} g_4(r) \end{vmatrix}. \tag{8}$$

Let us make inverse transformation to a non-cyclic presentation, $\Phi^c_j = S^{-1} \overline{\Phi}^c_j$:

$$\Psi^c_j = e^{-i\epsilon t} e^{ikz} \begin{vmatrix} \frac{1}{2}\left(-e^{i/2\phi} e^{ij\phi} g_2 + e^{-i3/2\phi} e^{ij\phi} g_4\right) \\ \frac{1}{2}\left(e^{+i3/2\phi} e^{ij\phi} g_1 - e^{-i/2\phi} e^{ij\phi} g_3\right) \\ e^{-i3/2\phi} e^{ij\phi} g_4 \\ e^{+i3/2\phi} e^{ij\phi} g_1 \end{vmatrix}. \tag{9}$$

Let us find four equations corresponding to the equation in non-diagonal basis:

$$\left[2M\epsilon + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\left(\frac{\partial}{\partial\phi} + ieA_\phi\right)^2 + \frac{\partial^2}{\partial z^2} + eBS_3 \right] \Psi^c_j = 0;$$

making the formal replacements $eB \Rightarrow B$, $eA \Rightarrow A$, $\partial^2/\partial z^2 \Rightarrow -k^2$, $2M\epsilon - k^2 \Rightarrow E$, we get

$$\left[E + \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\left(\frac{\partial}{\partial\phi} + iA_\phi\right)^2 + B \begin{vmatrix} -1/2 & 0 & 1 & 0 \\ 0 & +1/2 & 0 & -1 \\ 0 & 0 & +3/2 & 0 \\ 0 & 0 & 0 & -3/2 \end{vmatrix} \right] \begin{vmatrix} \left(-e^{i\sigma 2\phi} g_2 + e^{i\sigma 4\phi} g_4\right)/2 \\ \left(e^{i\sigma 1\phi} g_1 - e^{i\sigma 3\phi} g_3\right)/2 \\ e^{i\sigma 4\phi} g_4 \\ e^{i\sigma 1\phi} g_1 \end{vmatrix} = 0;$$

whence 4 separate equations follow (let us apply the shortening notation $\Delta = E + \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}$):

$$
\begin{gathered}
-\frac{1}{2}\left[\Delta-\frac{1}{r^2}(\sigma_2+A_\phi)^2-\frac{1}{2}B\right]e^{i\sigma 2\phi}g_2+\frac{1}{2}\left[\Delta-\frac{1}{r^2}(\sigma_4+A_\phi)^2+\frac{3}{2}B\right]e^{i\sigma 4\phi}g_4=0,\\
\frac{1}{2}\left[\Delta-\frac{1}{r^2}(\sigma_1+A_\phi)^2-\frac{3}{2}B\right]e^{i\sigma 1\phi}g_1-\frac{1}{2}\left[\Delta-\frac{1}{r^2}(\sigma_3+A_\phi)^2-\frac{1}{2}B\right]e^{i\sigma 3\phi}g_3=0,\\
\left[\Delta-\frac{1}{r^2}(\sigma_4+A_\phi)^2+\frac{3}{2}B\right]e^{i\sigma 4\phi}g_4=0,\qquad \left[\Delta-\frac{1}{r^2}(\sigma_1+A_\phi)^2-\frac{3}{2}B\right]e^{i\sigma 1\phi}g_1=0.
\end{gathered}
\tag{10}
$$

Let us take into account $\sigma_1=j+3/2, \sigma_2=j+1/2, \sigma_3=j-1/2, \sigma_4=j-3/2,$ then we get

$$
\begin{gathered}
-\left[\Delta-\frac{1}{r^2}(j+1/2+A_\phi)^2-\frac{1}{2}B\right]e^{i(j+1/2)\phi}g_2+\left[\Delta-\frac{1}{r^2}(j-3/2+A_\phi)^2+\frac{3}{2}B\right]e^{i(j-3/2)\phi}g_4=0,\\
\left[\Delta-\frac{1}{r^2}(j+3/2+A_\phi)^2-\frac{3}{2}B\right]e^{i(j+3/2)\phi}g_1-\left[\Delta-\frac{1}{r^2}(j-1/2+A_\phi)^2-\frac{1}{2}B\right]e^{i(j-1/2)\phi}g_3=0,\\
\left[\Delta-\frac{1}{r^2}(j-3/2+A_\phi)^2+\frac{3}{2}B\right]e^{i(j-3/2)\phi}g_4=0,\qquad \left[\Delta-\frac{1}{r^2}(j+3/2+A_\phi)^2-\frac{3}{2}B\right]e^{i(j+3/2)\phi}g_1=0;
\end{gathered}
\tag{11}
$$

in each equation, we can eliminate the total multiplier $e^{ij\phi}$, so obtaining

$$
\begin{gathered}
-\left[\Delta-\frac{1}{r^2}(j+1/2+A_\phi)^2-\frac{1}{2}B\right]e^{i(1/2)\phi}g_2+\left[\Delta-\frac{1}{r^2}(j-3/2+A_\phi)^2+\frac{3}{2}B\right]e^{i(-3/2)\phi}g_4=0,\\
\left[\Delta-\frac{1}{r^2}(j+3/2+A_\phi)^2-\frac{3}{2}B\right]e^{i(+3/2)\phi}g_1-\left[\Delta-\frac{1}{r^2}(j-1/2+A_\phi)^2-\frac{1}{2}B\right]e^{i(-1/2)\phi}g_3=0,\\
\left[\Delta-\frac{1}{r^2}(j-3/2+A_\phi)^2+\frac{3}{2}B\right]e^{i(-3/2)\phi}g_4=0,\qquad \left[\Delta-\frac{1}{r^2}\big((j+3/2)+A_\phi\big)^2-\frac{3}{2}B\right]e^{i(+3/2)\phi}g_1=0.
\end{gathered}
\tag{12}
$$

Allowing for the 3-rd and 4-th equations, we can derive 4 unlinked equations (in all equations, we can eliminate the ϕ-dependent multipliers):

$$
\begin{gathered}
\left[\Delta-\frac{1}{r^2}(j+3/2+A_\phi)^2-\frac{3}{2}B\right]g_1=0,\qquad \left[\Delta-\frac{1}{r^2}(j+1/2+A_\phi)^2-\frac{1}{2}B\right]g_2=0,\\
\left[\Delta-\frac{1}{r^2}(j+3/2+A_\phi)^2-\frac{3}{2}B\right]g_3=0,\qquad \left[\Delta-\frac{1}{r^2}(j-3/2+A_\phi)^2+\frac{3}{2}B\right]g_4=0.
\end{gathered}
\tag{13}
$$

This simple system refers to solutions Ψ_j^c in Cartesian tetrad basis and in the non-cyclic presentation of the spin matrix S_3; however, the corresponding substitution is not simple – see (8). In the cyclic presentation, we get the same four equations, but with the use of a more simple substitution (7).

Solving the 2nd order equations. We start with the equation in cyclic basis for states $\bar{\Psi}_j^c$:

$$
\bar{\Psi}^c=e^{-i\epsilon t}e^{ikz}\left|\begin{array}{c} e^{im\phi}g_1(r)\\ e^{im\phi}g_2(r)\\ e^{im\phi}g_3(r)\\ e^{im\phi}g_4(r)\end{array}\right|,\quad j=j_3\equiv m,
\tag{14}
$$

setting $c\Rightarrow 1,\ \hbar\Rightarrow 1,\ eB\Rightarrow B,$ we arrive at 4 unlinked equations:

$$
\epsilon g_1=-\frac{1}{2M}\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}\left(m-\frac{Br^2}{2}\right)^2-k^2\right]g_1+\frac{B}{2M}\sigma_1 g_1,
$$

$$
\epsilon g_2=-\frac{1}{2M}\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}\left(m-\frac{Br^2}{2}\right)^2-k^2\right]g_2+\frac{B}{2M}\sigma_2 g_2,
$$

$$\begin{aligned}\epsilon g_3 &= -\frac{1}{2M}\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}\left(m-\frac{Br^2}{2}\right)^2-k^2\right]g_3+\frac{B}{2M}\sigma_3 g_3,\\ \epsilon g_4 &= -\frac{1}{2M}\left[\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}-\frac{1}{r^2}\left(m-\frac{Br^2}{2}\right)^2-k^2\right]g_4+\frac{B}{2M}\sigma_4 g_4.\end{aligned} \tag{15}$$

In the vicinity of the point $r = 0$, we get $g_k = r^{a_k}$, $a_k = +|m|, -|m|$; to describe bound states, we should use the following asymptotic $r = r^{+|m|}$. For dimensionless variable $x = Br^2/2$, equations take the form

$$\left[2M\epsilon+2Bx\frac{d^2}{dx^2}+2B\frac{d}{dx}-\frac{B}{2x}(m-x)^2-k^2+\frac{3B}{2}\right]g_1=0,$$

$$\left[2M\epsilon+2Bx\frac{d^2}{dx^2}+2B\frac{d}{dx}-\frac{B}{2x}(m-x)^2-k^2+\frac{B}{2}\right]g_2=0,$$

$$\left[2M\epsilon+2Bx\frac{d^2}{dx^2}+2B\frac{d}{dx}-\frac{B}{2x}(m-x)^2-k^2-\frac{B}{2}\right]g_3=0,$$

$$\left[2M\epsilon+2Bx\frac{d^2}{dx^2}+2B\frac{d}{dx}-\frac{B}{2x}(m-x)^2-k^2-\frac{3B}{2}\right]g_4=0;$$

they have the same general structure

$$\left[x\frac{d^2}{dx^2}+\frac{d}{dx}-\frac{1}{4}\left(\frac{m^2}{x}+x-2m\right)+\left(\frac{2M\epsilon-k^2}{2B}-\frac{\sigma}{2}\right)\right]g=0; \tag{16}$$

note the physical dimensions of the quantities:

$$[x]=1,\quad [M]=\frac{1}{L},\quad [k^2]=\frac{1}{L^2},\quad [2M\epsilon]=\frac{1}{L^2},\quad [B]=\frac{1}{L^2}.$$

Let us apply the shortening notations

$$\frac{2M\epsilon-k^2}{2B}-\frac{\sigma}{2}=\beta,\qquad \left[x\frac{d^2}{dx^2}+\frac{d}{dx}-\frac{1}{4}\left(\frac{m^2}{x}+x-2m\right)+\beta\right]g=0. \tag{17}$$

In the vicinity of the point $x = 0$, the above equation gives $g = x^{\alpha}$, $\alpha = +|m|/2,\ -|m|/2$; at $x \to \infty$ we get $g = e^{\lambda x}$, $\lambda = +1/2, -1/2$. Because $x = Br^2/2$, where $B > 0$, at r-infinity, the variable $x \to +\infty$; so the needed asymptotic is $g_\infty \sim e^{-x/2}$. Thus, searching for solutions in the form $g = x^{\alpha}e^{-x/2}G(x)$, we derive

$$xG''+(|m|+1-x)G'+\left(\frac{m-|m|}{2}-\frac{1}{2}+\beta\right)G=0.$$

This is an equation of hypergeometric type

$$x\frac{d^2G}{dx^2}+(c-x)\frac{dG}{dx}-aG=0,\quad c=|m|+1,\quad a=-\left(\frac{m-|m|}{2}-\frac{1}{2}+\beta\right).$$

Imposing the polynomial condition $a = -n$, $n = 0,1,2\ldots$, we get

$$\frac{m-|m|}{2}-\frac{1}{2}+\frac{2M\epsilon-k^2}{2B}-\frac{\sigma}{2}=n\Rightarrow\frac{2M\epsilon-k^2}{2B}=n+\frac{1}{2}+\frac{\sigma}{2}+\frac{|m|-m}{2}.$$

Thus, the transverse motion of the particle is quantized according to the rule

$$2M\epsilon-k^2=2B\left(n_s+\frac{\sigma_s+1}{2}+\frac{|m|-m}{2}\right),\quad \sigma_s=-\frac{3}{2},-\frac{1}{2},\frac{1}{2},\frac{3}{2}. \tag{18}$$

In order to describe the energy level degeneration, we should use other notations (recall that $m = j_3 = j$)

$$2M\epsilon_N - k^2 = 2B\left(N + \frac{|m| - m}{2}\right), \quad N = n_s + \frac{\sigma_s + 1}{2}. \tag{19}$$

Now let us turn to solutions with fixed quantum numbers σ_n, $-i\partial_\phi \bar{\Phi}^c_n = \sigma_n \bar{\Phi}^c_n$. For this case, we have the substitution

$$\bar{\Phi}^c = e^{-i\epsilon t} e^{ikz} \begin{vmatrix} e^{i\sigma_1\phi} f_1(r) \\ e^{i\sigma_2\phi} f_2(r) \\ e^{i\sigma_3\phi} f_3(r) \\ e^{i\sigma_4\phi} f_4(r) \end{vmatrix}, \quad -i\partial_\phi \begin{vmatrix} e^{i\sigma_1\phi} f_1(r) \\ e^{i\sigma_2\phi} f_2(r) \\ e^{i\sigma_3\phi} f_3(r) \\ e^{i\sigma_4\phi} f_4(r) \end{vmatrix} = \begin{vmatrix} \sigma_1 e^{i\sigma_1\phi} f_1(r) \\ \sigma_2 e^{i\sigma_2\phi} f_2(r) \\ \sigma_3 e^{i\sigma_3\phi} f_3(r) \\ \sigma_4 e^{i\sigma_4\phi} f_4(r) \end{vmatrix}; \tag{20}$$

correspondingly, from the main equation we get 4 similar equations

$$\left[2M\epsilon - k^2 + \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\left(\sigma_n - \frac{Br^2}{2}\right)^2 - B\sigma_n\right] g_n = 0, \quad \sigma = -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}. \tag{21}$$

These equations belong to the above examined type, the difference consists only in the replacement of the parameter m by σ_n. Therefore, we derive the needed spectra by formal changes:

$$2M\epsilon_n - k^2 = 2B\left(n + \frac{1}{2} + \frac{\sigma_n}{2} + \frac{|\sigma_n| - \sigma_n}{2}\right), \quad \sigma = -\frac{3}{2}, -\frac{1}{2}, +\frac{1}{2}, +\frac{3}{2}. \tag{22}$$

Cylindrical tetrad basis, the gauge transformation. The task under consideration by means of a relevant gauge matrix may be transformed to cylindrical tetrad. To this end, let us recall some details from [11, 13], where the non-relativistic approximation for a spin 3/2 particle was studied. It was shown that if in the relativistic 16-component wave function for a spin 3/2 particle we preserve only the large components, as the result we obtain the function with a simpler structure; besides, in the paper [12] the relevant gauge transformation for six large components from Cartesian to cylindrical tetrad was derived:

$$\Phi^c = \begin{vmatrix} 0 & L_1^c & L_3^c & L_5^c \\ 0 & L_2^c & L_4^c & L_6^c \\ 0 & L_1^c & L_3^c & L_5^c \\ 0 & L_2^c & L_4^c & L_6^c \end{vmatrix}, \quad \Phi^c = \begin{vmatrix} L_1^c & L_3^c & L_5^c \\ L_2^c & L_4^c & L_6^c \end{vmatrix}, \quad \Phi^{\text{cyl}} = (B \otimes O)\Phi^c, \tag{23}$$

where the 2-spinor and 3-vector transformations are determined by the formulas

$$B = \begin{vmatrix} e^{+i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{vmatrix}, \quad O = \begin{vmatrix} \cos\phi & +\sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{vmatrix}. \tag{24}$$

Therefore, the needed gauge transformation is defined according to the rule

$$\Phi^{\text{cyl}} = (B \otimes O)\Phi^c = B \begin{vmatrix} L_1^c & L_3^c & L_5^c \\ L_2^c & L_4^c & L_6^c \end{vmatrix} \tilde{O} =$$

$$= \begin{vmatrix} e^{i\phi/2}\left(\cos\phi L_1^c + \sin\phi L_3^c\right) & e^{i\phi/2}\left(-\sin\phi L_1^c + \cos\phi L_3\right) & e^{i\phi/2} L_5^c \\ e^{-i\phi/2}\left(\cos\phi L_2^c + \sin\phi L_4^c\right) & e^{-i\phi/2}\left(-\sin\phi L_2^c + \cos\phi L_4^c\right) & e^{-i\phi/2} L_6^c \end{vmatrix}, \tag{25}$$

whence it follows

$$\begin{aligned}
&L_1^{\text{cyl}}=e^{i\phi/2}\left(\frac{1}{2}(e^{i\phi}+e^{-i\phi})L_1^c-\frac{i}{2}(e^{i\phi}-e^{-i\phi})L_3^c\right),\\
&L_2^{\text{cyl}}=e^{-i\phi/2}\left(\frac{1}{2}(e^{i\phi}+e^{-i\phi})L_2^c-\frac{i}{2}(e^{i\phi}-e^{-i\phi})L_4^c\right),\\
&L_3^{\text{cyl}}=e^{i\phi/2}\left(\frac{i}{2}(e^{i\phi}-e^{-i\phi})L_1^c+\frac{1}{2}(e^{i\phi}+e^{-i\phi})L_3^c\right),\\
&L_4^{\text{cyl}}=e^{-i\phi/2}\left(\frac{i}{2}(e^{i\phi}-e^{-i\phi})L_2^c+\frac{1}{2}(e^{i\phi}+e^{-i\phi})L_4^c\right),\\
&L_5^{\text{cyl}}=e^{i\phi/2}L_5^c,\quad L_6^{\text{cyl}}=e^{-i\phi/2}L_6^c.
\end{aligned}\tag{26}$$

According to [11, 13], only 4 large components L_1, L_2, L_5, L_6 are independent; two remaining are determined by the formulas $L_3^c=i(L_1^c-L_6^c)$, $L_4^c=-i(L_2^c+L_5^c)$. After eliminating the variables L_3^c, L_4^c, we obtain the transformation rule for 4 independent variables:

$$\begin{aligned}
&L_1^{\text{cyl}}=\frac{1}{2}(e^{3i\phi/2}+e^{-i\phi/2})L_1^c+\frac{1}{2}(e^{3i\phi/2}-e^{-i\phi/2})\left(L_1^c-L_6^c\right),\\
&L_2^{\text{cyl}}=\frac{1}{2}(e^{i\phi/2}+e^{-3i\phi/2})L_2^c+\frac{1}{2}(e^{i\phi/2}-e^{-3i\phi/2})\left(L_2^c+L_5^c\right),\\
&L_5^{\text{cyl}}=e^{i\phi/2}L_5^c,\qquad L_6^{\text{cyl}}=e^{-i\phi/2}L_6^c.
\end{aligned}\tag{27}$$

Instead of L_k^c, let us introduce more convenient variables Ψ^c, for which the structure of Pauli-like equation is given with the use of the third spin projection [8–10]:

$$\Psi^c=SL^c=\begin{vmatrix}\Psi_1^c\\ \Psi_2^c\\ \Psi_3^c\\ \Psi_4^c\end{vmatrix}=\begin{vmatrix}1&0&0&1\\0&1&-1&0\\2&0&0&-1\\0&2&1&0\end{vmatrix}\begin{vmatrix}L_1^c\\ L_2^c\\ L_5^c\\ L_6^c\end{vmatrix},\quad S^{-1}=\begin{vmatrix}1/3&0&1/3&0\\0&1/3&0&1/3\\0&-2/3&0&1/3\\2/3&0&-1/3&0\end{vmatrix},\tag{28}$$

$$\Psi_1^c=L_1^c+L_6^c,\quad \Psi_2^c=L_2^c-L_5^c,\quad \Psi_3^c=2L_1^c-L_6^c,\quad \Psi_4^c=2L_2^c+L_5^c;$$

the inverse transformation reads as

$$L_1^c=\frac{1}{3}\Psi_1^c+\frac{1}{3}\Psi_3^c,\quad L_2^c=\frac{1}{3}\Psi_2^c+\frac{1}{3}\Psi_4^c,\quad L_5^c=-\frac{2}{3}\Psi_2^c+\frac{1}{3}\Psi_4^c,\quad L_6^c=\frac{2}{3}\Psi_1^c-\frac{1}{3}\Psi_3^c.\tag{29}$$

For the variables Ψ_k^c, the formulas (27) take other form

$$\begin{aligned}
&L_1^{\text{cyl}}=\frac{1}{3}\Psi_1^c e^{-\frac{i\phi}{2}}-\frac{1}{6}\Psi_3^c e^{-\frac{i\phi}{2}}+\frac{1}{2}\Psi_3^c e^{\frac{3i\phi}{2}},\quad L_2^{\text{cyl}}=\frac{1}{3}\Psi_2^c e^{\frac{i\phi}{2}}-\frac{1}{6}\Psi_4^c e^{\frac{i\phi}{2}}+\frac{1}{2}\Psi_4^c e^{-\frac{3i\phi}{2}},\\
&L_5^{\text{cyl}}=\frac{1}{3}\Psi_4^c e^{\frac{i\phi}{2}}-\frac{2}{3}\Psi_2^c e^{\frac{i\phi}{2}},\quad L_6^{\text{cyl}}=\frac{2}{3}\Psi_1^c e^{-\frac{i\phi}{2}}-\frac{1}{3}\Psi_3^c e^{-\frac{i\phi}{2}}.
\end{aligned}\tag{30}$$

Taking into account (9) – where the multiplier $e^{-i\epsilon t}e^{ikz}$ is omitted – we can rewrite the relations (30) differently

$$\begin{aligned}
&L_1^{\text{cyl}}(r,\phi)=e^{ij\phi}\left[\frac{1}{2}g_4(r)-\frac{1}{6}g_2(r)\right],\quad L_2^{\text{cyl}}(r,\phi)=e^{ij\phi}\left[\frac{1}{2}g_1(r)-\frac{1}{6}g_3(r)\right],\\
&L_5^{\text{cyl}}(r,\phi)=\frac{1}{3}g_3(r)e^{ij\phi},\quad L_6^{\text{cyl}}(r,\phi)=-\frac{1}{3}g_2(r)e^{ij\phi},
\end{aligned}\tag{31}$$

the corresponding inverse transformation reads as

$$\begin{gathered} g_1(r) = \Big[2L_2^{\text{cyl}}(r,\phi) + L_5^{\text{cyl}}(r,\phi) \Big] e^{-ij\phi}, \quad g_2(r) = -3L_6^{\text{cyl}}(r,\phi)\, e^{-ij\phi}, \\ g_3(r) = 3L_5^{\text{cyl}}(r,\phi)\, e^{-ij\phi}, \quad g_4(r) = \Big[2L_1^{\text{cyl}}(r,\phi) - L_6^{\text{cyl}}(r,\phi) \Big] e^{-ij\phi}. \end{gathered} \tag{32}$$

In the cylindrical tetrad basis, the transformation to more convenient variables was performed:

$$\begin{gathered} L_1^{\text{cyl}}(r,\phi) = \frac{1}{3}\Big[\Psi_1^{\text{cyl}}(r,\phi) + \Psi_3^{\text{cyl}}(r,\phi) \Big], \quad L_6^{\text{cyl}}(r,\phi) = \frac{1}{3}\Big[2\Psi_1^{\text{cyl}}(r,\phi) - \Psi_3^{\text{cyl}}(r,\phi) \Big], \\ L_2^{\text{cyl}}(r,\phi) = \frac{1}{3}\Big[\Psi_2^{\text{cyl}}(r,\phi) + \Psi_4^{\text{cyl}}(r,\phi) \Big], \quad L_5^{\text{cyl}}(r,\phi) = \frac{1}{3}\Big[\Psi_4^{\text{cyl}}(r,\phi) - 2\Psi_2^{\text{cyl}}(r,\phi) \Big] \end{gathered} \tag{33}$$

and inverse formulas read as

$$\begin{gathered} \Psi_1^{\text{cyl}}(r,\phi) = L_1^{\text{cyl}}(r,\phi) + L_6^{\text{cyl}}(r,\phi), \quad \Psi_2^{\text{cyl}}(r,\phi) = L_2^{\text{cyl}}(r,\phi) - L_5^{\text{cyl}}(r,\phi), \\ \Psi_3^{\text{cyl}}(r,\phi) = 2L_1^{\text{cyl}}(r,\phi) - L_6^{\text{cyl}}(r,\phi), \quad \Psi_4^{\text{cyl}}(r,\phi) = 2L_2^{\text{cyl}}(r,\phi) + L_5^{\text{cyl}}(r,\phi). \end{gathered} \tag{34}$$

So we find the substitution for Ψ_j^{cyl}

$$\begin{aligned} & e^{-ij\phi}\Psi_1^{\text{cyl}}(r) = \frac{1}{2}g_4(r) - \frac{1}{6}g_2(r) - \frac{1}{3}g_2(r) = \frac{1}{2}g_4(r) - \frac{1}{2}g_2(r), \\ & e^{-ij\phi}\Psi_2^{\text{cyl}}(r) = \frac{1}{2}g_1(r) - \frac{1}{6}g_3(r) - \frac{1}{3}g_3(r) = \frac{1}{2}g_1(r) - \frac{1}{2}g_3(r), \\ & e^{-ij\phi}\Psi_3^{\text{cyl}}(r) = g_4(r) - \frac{1}{3}g_2(r) + \frac{1}{3}g_2(r) = g_4(r), \\ & e^{-ij\phi}\Psi_4^{\text{cyl}}(r) = g_1(r) - \frac{1}{3}g_3(r) + \frac{1}{3}g_3(r) = g_1(r); \end{aligned} \tag{35}$$

the inverse formulas are

$$\begin{gathered} g_1(r) = \Psi_4^{\text{cyl}}(r) = F_4 \equiv G_1, \quad g_2(r) = +\Psi_3^{\text{cyl}}(r) - 2\Psi_1^{\text{cyl}}(r) = F_3 - 2F_1 \equiv G_2, \\ g_3(r) = \Psi_4^{\text{cyl}}(r) - 2\Psi_2^{\text{cyl}}(r) = F_4 - 2F_2 \equiv G_3, \quad g_4(r) = \Psi_3^{\text{cyl}}(r) = F_3 \equiv G_4. \end{gathered} \tag{36}$$

We can transform the main equation from Cartesian tetrad basis

$$\left[2M\epsilon\Psi_c + \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} + \frac{1}{r^2}\left(\frac{\partial}{\partial\phi} + iBr^2/2 \right)^2 - k^2 + BS_3 \right]\Psi_j^c = 0; \tag{37}$$

to the cylindrical variables Ψ^{cyl} (see (36)):

$$\Psi_j^c = \left| \begin{matrix} -\frac{1}{2}e^{i(j+1/2)\phi}g_2 + \frac{1}{2}e^{i(j-3/2)\phi}g_4 \\ \frac{1}{2}e^{i(j+3/2)\phi}g_1 - \frac{1}{2}e^{i(j-1/2)\phi}g_3 \\ e^{i(j-3/2)\phi}g_4 \\ e^{i(j+3/2)\phi}g_1 \end{matrix} \right| = \left| \begin{matrix} -\frac{1}{2}e^{i(j+1/2)\phi}G_2 + \frac{1}{2}e^{i(j-3/2)\phi}G_4 \\ \frac{1}{2}e^{i(j+3/2)\phi}G_1 - \frac{1}{2}e^{i(j-1/2)\phi}G_3 \\ e^{i(j-3/2)\phi}G_4 \\ e^{i(j+3/2)\phi}G_1 \end{matrix} \right| = \Psi_j^{\text{cyl}}. \tag{38}$$

In fact, the structures in (38) coincides with each other, up to the formal replacement $g_i(r) \Rightarrow G_i(r)$. Therefore, the above resulting equations must be the same:

$$-\left[\Delta - \frac{1}{r^2}(j + 1/2 + A_\phi)^2 - \frac{1}{2}B \right] e^{i(1/2)\phi}G_2 + \left[\Delta - \frac{1}{r^2}(j - 3/2 + A_\phi)^2 + \frac{3}{2}B \right] e^{i(-3/2)\phi}G_4 = 0,$$

$$\left[\Delta - \frac{1}{r^2}(j + 3/2 + A_\phi)^2 - \frac{3}{2}B \right] e^{i(+3/2)\phi}G_1 - \left[\Delta - \frac{1}{r^2}(j - 1/2 + A_\phi)^2 - \frac{1}{2}B \right] e^{i(-1/2)\phi}G_3 = 0,$$

$$\left[\Delta-\frac{1}{r^2}(j-3/2+A_\phi)^2+\frac{3}{2}B\right]e^{i(-3/2)\phi}G_4=0,\quad \left[\Delta-\frac{1}{r^2}\big((j+3/2)+A_\phi\big)^2-\frac{3}{2}B\right]e^{i(+3/2)\phi}G_1=0. \tag{39}$$

Allowing for the 3-rd and 4-th equations, we can derive 4 unlinked equations:

$$\begin{gathered}\left[\Delta-\frac{1}{r^2}(j+3/2+A_\phi)^2-\frac{3}{2}B\right]G_1=0,\quad \left[\Delta-\frac{1}{r^2}(j+1/2+A_\phi)^2-\frac{1}{2}B\right]G_2=0,\\ \left[\Delta-\frac{1}{r^2}(j+3/2+A_\phi)^2-\frac{3}{2}B\right]G_3=0,\quad \left[\Delta-\frac{1}{r^2}(j-3/2+A_\phi)^2+\frac{3}{2}B\right]G_4=0.\end{gathered} \tag{40}$$

These equations refer to cylindrical tetrad basis (38) according to the rules:

$$\begin{gathered}G_1(r)=\Psi_4^{\text{cyl}}(r),\quad G_2(r)=\left[\Psi_3^{\text{cyl}}(r)-2\Psi_1^{\text{cyl}}(r)\right],\\ G_3(r)=\left[\Psi_4^{\text{cyl}}(r)-2\Psi_2^{\text{cyl}}(r)\right],\quad G_4(r)=\Psi_3^{\text{cyl}}(r),\\ \Psi_j^{\text{cyl}}=\begin{vmatrix}-\frac{1}{2}e^{i(j+1/2)\phi}\left[\Psi_3^{\text{cyl}}-2\Psi_1^{\text{cyl}}\right]+\frac{1}{2}e^{i(j-3/2)\phi}\,\Psi_3^{\text{cyl}}\\ \frac{1}{2}e^{i(j+3/2)\phi}\,\Psi_4^{\text{cyl}}-\frac{1}{2}e^{i(j-1/2)\phi}\left[\Psi_4^{\text{cyl}}-2\Psi_2^{\text{cyl}}\right]\\ e^{i(j-3/2)\phi}\,\Psi_3^{\text{cyl}}\\ e^{i(j+3/2)\phi}\,\Psi_4^{\text{cyl}}\end{vmatrix}.\end{gathered} \tag{41}$$

Transition to cyclic basis (in cylindrical tetrad) is reached as follows

$$\begin{gathered}\bar\Psi_1^{\text{cyl}}=e^{i(j+3/2)\phi}\,\Psi_4^{\text{cyl}},\quad \bar\Psi_2^{\text{cyl}}=e^{i(j+1/2)\phi}\left[\Psi_3^{\text{cyl}}-2\Psi_1^{\text{cyl}}\right],\\ \bar\Psi_3^{\text{cyl}}=e^{i(j-1/2)\phi}\left[\Psi_4^{\text{cyl}}-2\Psi_2^{\text{cyl}}\right],\quad \bar\Psi_4^{\text{cyl}}=e^{i(j-3/2)\phi}\,\Psi_3^{\text{cyl}}.\end{gathered} \tag{42}$$

Operator $\hat J_3^c$ and the gauge transformation to cylindrical tetrad. Starting with the formulas

$$\hat J^c=-i\partial_\phi+S_3,\quad S_3=\begin{vmatrix}-1/2 & 0 & 1 & 0\\ 0 & +1/2 & 0 & -1\\ 0 & 0 & +3/2 & 0\\ 0 & 0 & 0 & -3/2\end{vmatrix},\quad \Psi=\begin{vmatrix}\Psi_1^c\\ \Psi_2^c\\ \Psi_3^c\\ \Psi_4^c\end{vmatrix} \tag{43}$$

and taking into account relationships (30)

$$\begin{gathered}L_1^{\text{cyl}}(r,\phi)=\frac{1}{3}\Psi_1^c(r,\phi)e^{-\frac{i\phi}{2}}-\frac{1}{6}\Psi_3^c(r,\phi)e^{-\frac{i\phi}{2}}+\frac{1}{2}\Psi_3^c(r,\phi)e^{\frac{3i\phi}{2}},\\ L_2^{\text{cyl}}(r,\phi)=\frac{1}{3}\Psi_2^c(r,\phi)e^{\frac{i\phi}{2}}-\frac{1}{6}\Psi_4^c(r,\phi)e^{\frac{i\phi}{2}}+\frac{1}{2}\Psi_4^c(r,\phi)e^{-\frac{3i\phi}{2}},\\ L_5^{\text{cyl}}(r,\phi)=\frac{1}{3}\Psi_4^c(r,\phi)e^{\frac{i\phi}{2}}-\frac{2}{3}\Psi_2^c(r,\phi)e^{\frac{i\phi}{2}},\\ L_6^{\text{cyl}}(r,\phi)=\frac{2}{3}\Psi_1^c(r,\phi)e^{-\frac{i\phi}{2}}-\frac{1}{3}\Psi_3^c(r,\phi)e^{-\frac{i\phi}{2}},\end{gathered} \tag{44}$$

we should examine the following relations

$$\begin{gathered}(-i\partial_\phi+S_3)\Psi^c(r,\phi)=j\Psi^c(r,\phi),\quad L^{\text{cyl}}(r,\phi)=S(\phi)\Psi^c(r,\phi),\\ \left[S(-i\partial_\phi+S_3)S^{-1}\right]L^{\text{cyl}}(r,\phi)=j\,L^{\text{cyl}}(r,\phi);\end{gathered}$$

whence it follows $S(\phi)(-i\partial_\phi + S_3)S^{-1}(\phi) = \hat{J}_3^{\text{cyl}}$, so we get the transformation rule

$$\hat{J}_3^{\text{cyl}} = -i\partial_\phi - iS\left(\frac{\partial}{\partial\phi}S^{-1}\right) + SS_3S^{-1}, \tag{45}$$

where the matrix $S(\phi)$ is determined by (44), so that

$$S = \begin{vmatrix} \frac{1}{3}e^{-\frac{i\phi}{2}} & 0 & \left(-\frac{1}{6}e^{-\frac{i\phi}{2}} + \frac{1}{2}e^{\frac{3i\phi}{2}}\right) & 0 \\ 0 & \frac{1}{3}e^{\frac{i\phi}{2}} & 0 & \left(-\frac{1}{6}e^{\frac{i\phi}{2}} + \frac{1}{2}e^{-\frac{3i\phi}{2}}\right) \\ 0 & -\frac{2}{3}e^{\frac{i\phi}{2}} & 0 & \frac{1}{3}e^{\frac{i\phi}{2}}\Psi_4^c \\ \frac{2}{3}e^{-\frac{i\phi}{2}} & 0 & -\frac{1}{3}e^{-\frac{i\phi}{2}} & 0 \end{vmatrix}.$$

After performing the needed calculation, from (45) we get

$$\hat{J}_3^{\text{cyl}} = -i\partial_\phi + \begin{vmatrix} -3/2 & 0 & 0 & 1 \\ 0 & 3/2 & 1 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{vmatrix} + \begin{vmatrix} 3/2 & 0 & 0 & -1 \\ 0 & -3/2 & -1 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & -1/2 \end{vmatrix} = -i\partial_\phi.$$

Therefore, we arrive at the equation

$$-i\partial_\phi L_j^{\text{cyl}}(r,\phi) = j\, L_j^{\text{cyl}}(r,\phi) \Rightarrow -i\partial_\phi \Psi_j^{\text{cyl}}(r,\phi) = j\, \Psi_j^{\text{cyl}}(r,\phi). \tag{46}$$

Without additional calculation one can find the tetrad gauge transformation for orbital momentum $\hat{l}_3^c = -i\partial_\phi$ in the non-diagonal presentation:

$$\Psi, \quad \hat{l}_3^{\text{cyl}} = -i\partial_\phi - iS\left(\frac{\partial}{\partial_\phi}S^{-1}\right) = -i\partial_\phi + \begin{vmatrix} -3/2 & 0 & 0 & 1 \\ 0 & 3/2 & 1 & 0 \\ 0 & 0 & -1/2 & 0 \\ 0 & 0 & 0 & 1/2 \end{vmatrix}; \tag{47}$$

correspondingly, in the cyclic (diagonal) basis $\bar{\Psi}$ we get

$$\bar{\Psi}, \quad \hat{l}_3^{\text{cyl}} = -i\partial_\phi + \begin{vmatrix} -3/2 & 0 & 0 & 0 \\ 0 & -1/2 & 0 & 0 \\ 0 & 0 & +1/2 & 0 \\ 0 & 0 & 0 & +3/2 \end{vmatrix}. \tag{48}$$

Conclusion. In the present paper, we have examined the non-relativistic problem of a spin 3/2 particle in the external magnetic field, applying two tetrads: Cartesian and cylindrical, following the concomitant local spin-vector gauge transformation $B(\phi) \otimes O(\phi)$. We have detailed the tetrad gauge transformations for diagonalized operators of the total angular momentum and the orbital momentum $\hat{J}_3^{\text{cart}}, \hat{l}_3^{\text{cart}} \Rightarrow \hat{J}_3^{\text{cyl}}, \hat{l}_3^{\text{cyl}}$. Equivalent substitutions for the wave functions in Cartesian and cylindrical tetrads are found, for diagonal and non-diagonal bases. The second order equations arising in the non-relativistic problem are solved in terms of the confluent hypergeometric functions, and the corresponding energy spectra are found.

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