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ON THE TENSOR ANALYZING POWER COMPONENT T_{20} IN THE REACTION $\gamma d \to \pi^0 d$

Abstract. We calculate the tensor analyzing power component T_{20} for coherent photoproduction of pions on the deuteron in the framework of the impulse approximation. The sensitivity of the results to the choice of the deuteron wave function is investigated. We compare the obtained values with the predictions of other models and with experimental data. The noticeable discrepancy between the theory and experiment was found at the photon energy E_γ = 400 MeV.

Keywords: meson production, photoproduction reactions, few-body systems, deuteron, polarization phenomena in reactions, spin observables, polarized targets

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О КОМПОНЕНТЕ ТЕНЗОРНОЙ АНАЛИЗИРУЮЩЕЙ СПОСОБНОСТИ T_{20} В РЕАКЦИИ $\gamma d \to \pi^0 d$

Аннотация. В рамках плосковолнового импульсного приближения рассчитана компонента тензорной анализирующей способности T_{20} в реакции когерентного фоторождения пионов на дейтроне. Исследована чувствительность результатов к выбору дейтронной волновой функции. Проведено сравнение полученных результатов с предсказаниями других моделей и с экспериментальными данными. Наблюдается заметное расхождение между теорией и экспериментом при энергии фотона E_γ = 400 МэВ.

Ключевые слова: рождение мезонов, реакции фоторождения, системы нескольких тел, дейтрон, поляризационные явления в реакциях, спиновые наблюдаемые, поляризованные мишени

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Introduction. Experiments on the pion photoproduction on nucleons and nuclei provide a unique tool for understanding the internal structure of the nucleons and other strongly interacting systems. The deuteron is the simplest nucleus that contains the neutron. Its low binding energy and well-known structure in comparison with heavier nuclei gives great opportunity to study the neutron that is absent for the study as a free target. In this context, the deuteron researches are of particular interest.

The history of theoretical investigation of the coherent reaction on the deuteron began with the works [1, 2], where an approximate treatment of Fermi motion was used. Many theoretical studies using more delicate approaches have been carried out later. Among them are: studies of factorization approximation to treat the Fermi motion in impulse approximation (IA) [3], different approaches with the final state rescattering [4–11], dynamical models for nucleon interactions with resonances [12].

In 2020, the data for unique measurements of the tensor analyzing power component T_{20} were published in Ref. [13]. The measurements were performed at VEPP-3 electron storage ring at the Budker Institute of Nuclear Physics in Novosibirsk, Russia.

In the present paper we investigate the capabilities of various models in reproducing experimental data [13]. Tests on polarization observables impose even more stringent requirements than data on unpolarized reaction. We construct our approach to calculate the tensor analyzing power component T_{20} in the reaction $\gamma d \to \pi^0 d$. We use a realistic pion photoproduction operator, based on unitary isobar model MAID07 [14], and parameterizations of the deuteron wave function (DWF) obtained from high-precision *NN* potentials. Note that in Ref. [13] our results on T_{20} were already shown but they were cited as a private communication by M. Levchuk dated on 2013. In the present paper we give some details of the calculations.

1. Kinematics and observables. In the present work, we consider the reaction $\gamma d \to \pi^0 d$ in the photon-deuteron (γd) center-of-mass (c.m.) frame. Let us denote by $k = (\omega, \mathbf{k})$, $p = (E, -\mathbf{k})$, $q = (\varepsilon_\pi, \mathbf{q})$, and $p' = (E', -\mathbf{q})$ the four-momenta of the initial photon, initial deuteron, final pion, and final deuteron, respectively. We use a coordinate system in which z-axis is directed along the photon momentum $\mathbf{k}$ and y-axis is parallel to $\mathbf{k} \times \mathbf{q}$. A symbol E_γ stands for the photon energy in the laboratory frame: $\omega = E_\gamma M/W$ with $W = \sqrt{M^2 + 2ME_\gamma} = \omega + \sqrt{M^2 + \omega^2}$ and M being the deuteron mass. Note, that with the π^0 mass $\mu = 135.0$ MeV the reaction threshold is given by

$$E_\gamma^{th} = \mu + \frac{\mu^2}{2M} = 139.9 \text{ MeV}. \quad (1)$$

The pion c.m. momentum is

$$q = \frac{1}{2W}\sqrt{[W^2 - (M+\mu)^2][W^2 - (M-\mu)^2]}. \quad (2)$$

The c.m. emission angle for the pion is θ. Let m_d, m'_d, and λ be spin states of the initial deuteron, final deuteron, and photon, respectively. In practice, we calculated only a half of 18 amplitudes $\langle m'_d | T | \lambda m_d \rangle$, namely those for $\lambda = 1$. One has due to the parity conservation [6] that

$$\langle -m'_d | T | -\lambda - m_d \rangle = (-1)^{+m_d + m'_d + \lambda} \langle m'_d | T | \lambda m_d \rangle. \quad (3)$$

In principle, in the case of real photons ($\lambda = \pm 1$) this relation can be rewritten in a simpler form

$$\langle -m'_d | T | -\lambda - m_d \rangle = (-1)^{m_d + m'_d} \langle m'_d | T | \lambda m_d \rangle. \quad (4)$$

The tensor analyzing power component T_{20} is defined as

$$T_{20} = \frac{1}{\sqrt{2}} \sum_{m_d} \Big(|\langle m_d | T | +1+1 \rangle|^2 + |\langle m_d | T | +1-1 \rangle|^2 - 2|\langle m_d | T | +10 \rangle|^2 \Big) \Big/ \sum_{m'_d m_d} |\langle m'_d | T | +1 m_d \rangle|^2 . \quad (5)$$

2. The *T*-Matrix. We restrict ourselves in the present work to the impulse approximation (IA). The corresponding diagram is shown in Fig. 1.

The reaction amplitude in the IA is given by

$$\langle m'_d | \mathcal{M}(\mathbf{q}, \mathbf{k}) | 1 m_d \rangle = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{m_2 m_1 m} \Psi^{m'_d \dagger}_{m_2 m}\left(\mathbf{p} + \frac{\mathbf{q}}{2}\right) \langle m_2 | t_{\gamma\pi} | 1 m_1 \rangle \Psi^{m_d}_{m_1 m}\left(\mathbf{p} + \frac{\mathbf{k}}{2}\right), \quad (6)$$

where $t_{\gamma\pi}$ is the amplitude of π^0 photoproduction on the nucleon and Ψ^{m_d} denotes DWF. The scattering matrix $\mathcal{M}_{m'_d 1 m_d}$ is given by isolating the azimuthal dependence as follows

$$\mathcal{M}_{m'_d 1 m_d}(\Theta, \phi) = e^{i(1+m_d)\phi} T_{m'_d 1 m_d}(\Theta), \quad (7)$$

where the reduced *T*-matrix elements are defined by separating the ϕ-dependence from the $\mathcal{M}$-matrix elements and it is used in Eq. (5) to calculate T_{20}.

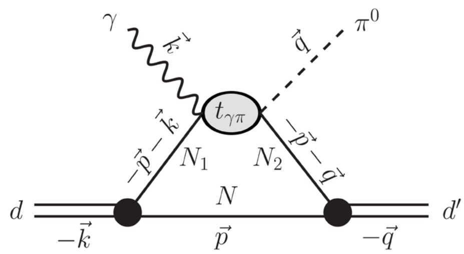

Fig. 1. Diagrammatic representation of the $\gamma d \to \pi^0 d$ amplitude in IA

The pion photoproduction operator $t_{\gamma\pi}$ in the photon-nucleon c.m. frame has the well-known form [15]

$$\langle m_2 \,|\, t_{\gamma\pi} \,|\, \lambda m_1 \rangle = \frac{4\pi W_{\gamma N_1}}{m} \langle m_2 \,|\, i(\boldsymbol{\sigma}\cdot\epsilon_\lambda)F_1 + (\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})\Big(\boldsymbol{\sigma}\cdot(\hat{\mathbf{k}}\times\epsilon_\lambda)\Big)F_2 + \\ + i(\boldsymbol{\sigma}\cdot\hat{\mathbf{k}})(\hat{\mathbf{q}}\cdot\epsilon_\lambda)F_3 + i(\boldsymbol{\sigma}\cdot\hat{\mathbf{q}})(\hat{\mathbf{q}}\cdot\epsilon_\lambda)F_4 \,|\, m_1 \rangle. \quad (8)$$

The amplitudes $F_i\,(i = 1-4)$ are taken from the MAID07 model for pion photoproduction on the nucleon [14]. Before using Eq. (8) one has to transform the unit vectors $\hat{\mathbf{q}}$ and $\hat{\mathbf{k}}$ and the polarization vector ϵ_λ to the γN_1 c.m. frame. This can be done making the use of a boost with the velocity $\mathbf{v} = \mathbf{p}\,/\,(E - \sqrt{m^2 + \mathbf{p}^2})$ with m being the nucleon mass.

3. Results and discussion. Figure 2 shows a comparison of results for the angular dependencies of T_{20} at three photon energies using different parameterizations of DWF. Black, blue and red curves are the results obtained using the wave functions corresponding to the potentials CD-Bonn [16], V18 [17] and Nijm93 [18], respectively. The results obtained for the potential NijmII [18] coincide with those for V18 and are not shown in the Figure. As one can see, the difference between the predictions is small. This is due to the fact that realistic nucleon-nucleon potentials are used in calculations. Yet we consider CD-Bonn *NN* potential as the main potential in this work. The χ^2/N_{dof} values are equal to 12.2, 13.8, 12.8, 13.8 for CD-Bonn, V18, Nijm93 and NijmII, respectively. If the data for E_γ = 400 MeV are not taken into account, the χ^2/N_{dof} values are equal to 8.6, 4.4, 4.8, 4.3.

Figure 3 shows the predictions for the angular dependencies of T_{20} at three photon energies made with the use of various theoretical models. Black curves as in Figure 2 are obtained in the framework of the model described in Section 1. Blue curves are the full calculation from work [7]. Red curves are obtained from a model similar to ours [8, 9]. Cyan lines are the predictions from work [6]. One can see that our model successfully describes the experimental data for the energies 260 and 340 MeV, but there is a large discrepancy for photon energy 400 MeV. Moreover, it follows from Figures 2 and 3 that none of the considered models describes the experimental data at the energy of 400 MeV. It should be noted that the discrepancy between our calculation and calculations [8, 9] is a separate important issue, since the models within which the calculations were carried out are identical.

Figures 4 and 5 show energy dependencies of T_{20} for fixed angles. Notations as in Figures 2 and 3. Green lines are calculations from Refs. [13]. One can see that our predictions have some small discrepancies with the experimental data at θ = 110°. For θ = 130° we have good agreement with the experiment up to 325 MeV, but our model does not describe the experimental data for higher energies.

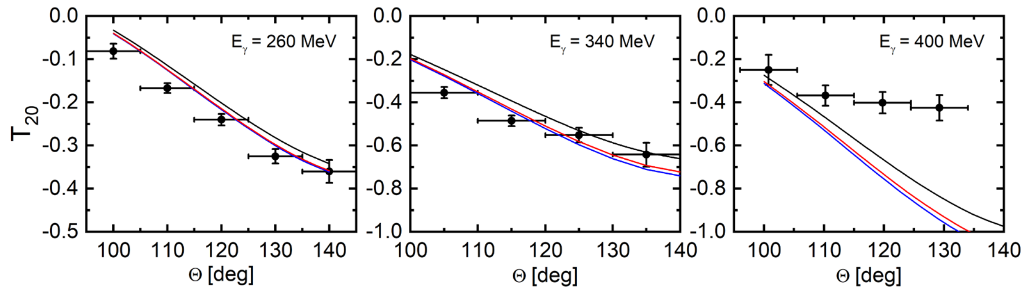

Fig. 2. The tensor analyzing power component T_{20} for various parameterizations of the deuteron wave function. Blue, red, and black lines correspond to DWF from Refs. [17, 18], and [16], respectively. Data are from Ref. [13]

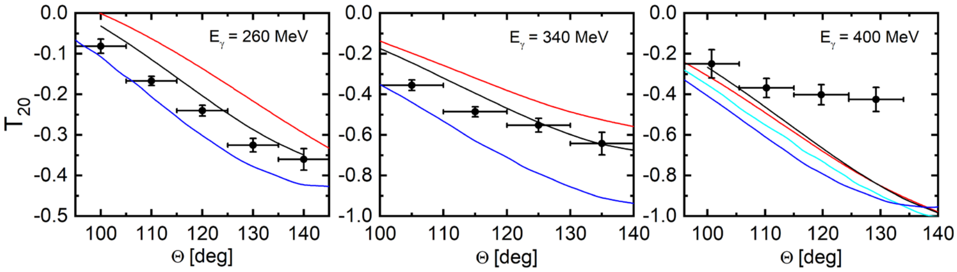

Fig. 3. Predictions of different models for the tensor analyzing power component T_{20}.
Cyan, blue, red and black lines are the results from Refs. [6–9], and the present work, respectively. Data are from Ref. [13]

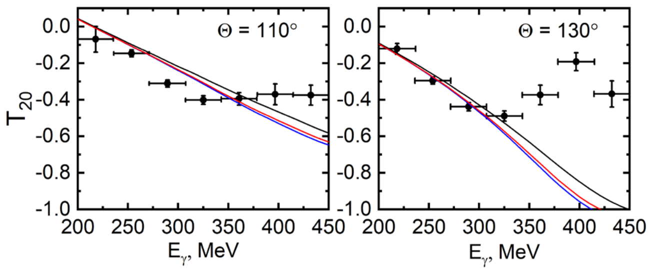

Fig. 4. The tensor analyzing power component T_{20} for various parameterizations of the deuteron wave function.
Blue, red, and black lines correspond to DWF from Refs. [17, 18] and [16], respectively. Data are from Ref. [13]

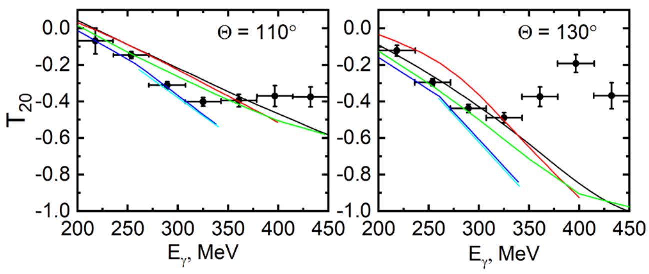

Fig. 5. Predictions of different models for the tensor analyzing power component T_{20}. Cyan, blue, red, green, and black lines are the results from Refs. [6–9, 13], and the present work, respectively. Data are from Ref. [13]

Conclusions. In the framework of the plane-wave impulse approximation using the unitary isobaric model MAID07 and various parameterizations of the deuteron wave function based on realistic nucleon-nucleon potentials, the tensor asymmetry component T_{20} in the reaction of coherent photoproduction of neutral pions on the deuteron in the framework of the impulse approximation is obtained. We have shown that the difference between predictions based on different *NN* potentials is small. Comparison of our model's predictions with experimental data shows that our model adequately describes the data for photon energies of 260 MeV and 340 MeV, but fails for energies of 400 MeV. It should be noted that all models presented in the literature have predictions close to ours at 400 MeV and, accordingly, the same problems. We do not have yet an explanation for this discrepancy.

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