ISSN 1561-2430 (Print)
ISSN 2524-2415 (Online)
UDC 621.396.96:519.246.8
https://doi.org/10.29235/1561-2430-2026-62-3-253-264

Received 25.08.2026
Поступила в редакцию 25.08.2026

Petr A. Khmarski[1], Valentin M. Artemiev[1], Alexander O. Naumov[1], Sergey V. Tsuprik[2], Bobomurod M. Muxammedov[3], Davron A. Juraev[3]

[1]Institute of Applied Physics of the National Academy of Sciences of Belarus, Minsk, Republic of Belarus
[2]Military Academy of the Republic of Belarus, Minsk, Republic of Belarus
[3]Institute of Military Aviation of the University of Military Security and Defense of the Republic of Uzbekistan, Qarshi, Republic of Uzbekistan

CORRECTION OF THE FILTERED FULL VELOCITY VECTOR USING AN INDEPENDENT ESTIMATE OF ITS RADIAL COMPONENT IN A MULTIFUNCTION RADAR

Abstract. The paper addresses the problem of improving the accuracy of the course and horizontal speed magnitude estimates produced by coordinate filtering in a multifunction radar operating in continuous-tracking mode with a high data-output rate, where the per-update object displacement is comparable to the single-measurement coordinate error. Under these conditions, the course and speed estimates obtained by differentiating noisy coordinates fluctuate from scan to scan, whereas the radial velocity (range rate), measured directly by the Doppler method, remains stable. A method is proposed for correcting the filtered full velocity vector estimate using an independently filtered radial-velocity estimate: the velocity vector is projected onto the constraint imposed by the radial velocity, with weights given by the variances of the coordinate filter; the course is reconciled with an independent geometric estimate from the track; the result is smoothed with inertia. The method does not change the coordinate estimate. Monte Carlo simulation (2000 runs per scenario) on three typical scenarios – helicopter, airplane, ballistic object – shows that the method reduces the course RMSE by a factor of 2.1 and the horizontal-speed-magnitude RMSE by a factor of 1.8–2.9 for the maneuvering objects; for the ballistic object it gives no benefit in course, which is already accurate, but reduces the speed-magnitude RMSE by a factor of 1.6 owing to a residual mismatch between the constant-acceleration coordinate filter and the true, non-constant deceleration. The method's applicability condition and the gain's sensitivity to update period, observation geometry, measurement accuracy, and maneuver intensity are examined separately.

Keywords: trajectory processing, radial velocity, range rate, Kalman filter, multifunction radar, velocity vector correction, statistical modeling, Monte Carlo method, root-mean-square error

For citation. Khmarski P. A., Artemiev V. M., Naumov A. O., Tsuprik S. V., Muxammedov B. M., Juraev D. A. Correction of the filtered full velocity vector using an independent estimate of its radial component in a multifunction radar. *Vestsi Natsyyanal'nai akademii navuk Belarusi. Seryya fizika-matematychnykh navuk = Proceedings of the National Academy of Sciences of Belarus. Physics and Mathematics series*, 2026, vol. 62, no. 3, pp. 253–264. https://doi.org/10.29235/1561-2430-2026-62-3-253-264

П. А. Хмарский[1], В. М. Артемьев[1], А. О. Наумов[1], С. В. Цуприк [2], Б. М. Мухаммедов[3], Д. А. Жураев[3]

[1]Институт прикладной физики Национальной академии наук Беларуси, Минск, Республика Беларусь
[2]Военная академия Республики Беларусь, Минск, Республика Беларусь
[3]Институт военной авиации Университета военной безопасности и обороны Республики Узбекистан, Карши, Республика Узбекистан

КОРРЕКЦИЯ РЕЗУЛЬТАТА ФИЛЬТРАЦИИ ПОЛНОГО ВЕКТОРА СКОРОСТИ ПО НЕЗАВИСИМОЙ ОЦЕНКЕ ЕЕ РАДИАЛЬНОЙ СОСТАВЛЯЮЩЕЙ В МНОГОФУНКЦИОНАЛЬНОМ РАДИОЛОКАТОРЕ

Аннотация. Рассматривается задача снижения погрешности фильтрации полного вектора скорости в многофункциональном радиолокаторе с высоким темпом выдачи данных, при котором объект за один период обновления смещается на величину, сопоставимую с разовой погрешностью измерения координат. В этих условиях оценки курса и скорости, получаемые дифференцированием зашумленных координат, колеблются от такта к такту, тогда как радиальная скорость, измеряемая доплеровским способом существенно точнее, остается устойчивой. Предложен метод коррекции результата фильтрации полного вектора скорости по независимо отфильтрованной оценке радиальной скорости: вектор скорости проецируется на ограничение, задаваемое радиальной скоростью, с весами по дисперсии фильтра координат, курс согласуется с независимой геометрической оценкой по треку, результат инерционно сглаживается. Метод не изменяет оценку координат. Работоспособность метода проверена методом Монте-Карло (2000 реализаций на сценарий) на трех типовых сценариях движения (вертолет, самолет, баллистический объект). Для маневрирующих объектов коррекция снижает среднеквадратическую погрешность (СКП) измерения курса в 2,1 раза и СКП измерения модуля горизонтальной скорости в 1,8–2,9 раза; для баллистического объекта выигрыша по курсу нет (он и без коррекции точен), но СКП измерения модуля горизонтальной скорости снижается в 1,6 раза за счет остаточного рассогласования модели постоянного ускорения с истинным, непостоянным по модулю торможением. Условие применимости метода, связанное с геометрической чувствительностью радиальной скорости к изменению модуля скорости, рассмотрено отдельно, наряду с чувствительностью выигрыша к периоду обновления, геометрии наблюдения, точности измерения радиальной скорости и интенсивности маневра.

Ключевые слова: траекторная обработка, радиальная скорость, фильтр Калмана, многофункциональный радиолокатор, коррекция вектора скорости, статистическое моделирование, метод Монте-Карло, среднеквадратическая погрешность

Для цитирования. Коррекция результата фильтрации полного вектора скорости по независимой оценке ее радиальной составляющей в многофункциональном радиолокаторе / П. А. Хмарский, В. М. Артемьев, А. О. Наумов [и др.] // Весці Нацыянальнай акадэміі навук Беларусі. Серыя фізіка-матэматычных навук. – 2026. – Т. 62, № 3. – С. 253–264. https://doi.org/10.29235/1561-2430-2026-62-3-253-264

Introduction. The development of multifunction radars (MFRs) with electronically scanned antennas and continuous object tracking increases the requirements for the accuracy and stability of estimating the motion parameters of observed objects [1]. Unlike radars with mechanical circular scanning, where tracking alternates with search and the object data update period is determined by the antenna rotation time, a multifunction radar is able to sustain a high rate of measurements of coordinates and radial velocity for a tracked object practically continuously; the allocation of the time and energy resources of such a radar among objects and modes is itself the subject of active research [2–4].

A high data-output rate, however, gives rise to a specific tracking-filter problem: over a single update period the object displacement is comparable to, or smaller than, the single-measurement error in coordinates. Under these conditions, the course and horizontal speed magnitude reconstructed by the coordinate tracking filter (for example, a Kalman filter or an adaptive filter with variable structure of the Interacting Multiple Model (IMM) type [5]) from a sequence of noisy coordinate measurements fluctuate from scan to scan – the useful displacement signal is comparable in magnitude to the measurement noise. For IMM filters this manifests, in particular, as instability (fluctuation) of model probabilities when hypothesis likelihoods are close, to counter which adaptive schemes for correcting the transition probability matrix have been proposed [6; 7]. The radial velocity V_r (range rate), measured by a multifunction radar directly (by the Doppler method) and an order of magnitude more accurately than coordinate differentiation allows, remains substantially more stable when filtered independently. This creates a prerequisite for correcting the velocity vector of the coordinate filter using an independently filtered radial-velocity estimate.

A problem that is essentially related (reconstructing the velocity vector from its radial component) is also actively studied in adjacent fields not connected with air-object tracking: in problems of Doppler odometry using 4D radars [8], tracking moving objects from 4D-radar point cloud using the measured radial velocity [9; 10], velocity vector estimation in automotive radar networks [11], and heading-constrained filtering for a high-frequency surface wave radar using the measured radial velocity [12]. This shows that using the radial velocity to refine the full velocity vector is a general and practically significant problem in radar information processing, rather than a specific feature of any particular radar architecture.

The authors have previously investigated related trajectory-processing problems: tracking of ground objects using onboard optical location system data [13], the main problems of trajectory processing in multiobject tracking [14], tracking processing devices for the indicator channels of short-range radio navigation systems [15], reducing radar measurement errors using ionosphere and magnetosphere monitoring data [16], a generalized technique for optimizing the parameters of tracking estimators [17], statistical synthesis of a Bayesian algorithm for measuring coordinates from image segmentation data [18], a systematic treatment of methods for processing measurements of coordinates and motion parameters in air and ground situation monitoring systems [19], and verification of the physical feasibility of a trajectory from surveillance radar data by numerical Monte Carlo integration [20].

This paper proposes and investigates a method of such correction in a generalized formulation: a hypothetical multifunction radar is considered, tracking an object in a single mode (without separation into search and track) with a constantly high data-output rate. The method's performance and the accuracy gain achieved in course and speed magnitude are shown from the results of statistical simulation by the Monte Carlo method on synthetic data for three typical object motion scenarios: a low-speed helicopter, an airplane, and a ballistic object.

Problem Statement. This paper considers the tracking system of a multifunction radar that forms, on each update cycle, a measurement consisting of range r, azimuth β, elevation angle ε, and radial velocity V_r with mutually independent measurement errors, converted to rectangular coordinates x, y, z by a standard transformation. The coordinate tracking filter (in the simplest case, a Kalman filter with a nearly constant-velocity motion model) provides, on each cycle, an estimate of the state vector including the coordinates and the velocity vector $\mathbf{V} = (V_x, V_y, V_z)^T$, together with the error covariance matrix of filter [21]. An independent filter for the radial velocity operates in parallel, providing an estimate $V_{r,\text{filtr}}$ and its variance.

The condition under which the course and horizontal speed magnitude estimates of the coordinate filter turn out to be substantially less stable than $V_{r,\text{filtr}}$ is formulated as smallness of the object's displacement over a single update period T relative to the single-measurement root-mean-square coordinate error σ:

$$\left|\mathbf{V}\right| T \le \sigma. \quad (1)$$

When (1) holds, the useful displacement signal is comparable in magnitude to the coordinate measurement noise, and the estimate of course and speed magnitude obtained from a sequence of coordinate measurements contains substantial variance. The task is to construct a working velocity vector $\mathbf{V}'$ that: (a) is exactly consistent with the independently measured and filtered radial velocity, (*b*) is robust to the noise of individual measurements, (c) does not require changing the coordinate estimate itself or the covariance of the coordinate filter.

Figure 1 illustrates the observation geometry for three scenarios used throughout this paper: the true horizontal-plane trajectory and altitude profile of a low-speed helicopter, an airplane, and a ballistic object, together with one realization of the noisy single measurements from which the coordinate filter must reconstruct the motion. The scatter of the individual measurements around the true trajectory is the geometric expression of condition (1): over a single update period the useful object displacement is comparable to this scatter, so the coordinate filter alone cannot reliably separate signal from noise.

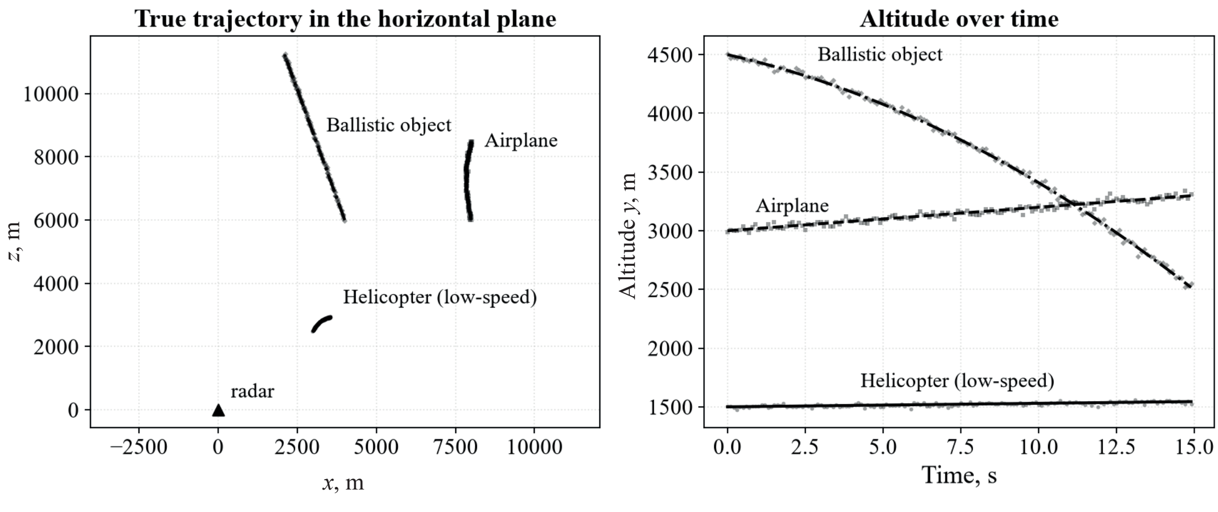


Fig. 1. Observation geometry for three scenarios – true horizontal-plane trajectory and altitude profile, with one realization of the noisy single measurements (radar at the origin)

Method of Velocity Vector Correction. Let **V** be the velocity vector estimate of the coordinate filter with diagonal covariance-matrix $\mathrm{diag}\left(\sigma_{V_x}^2, \sigma_{V_y}^2, \sigma_{V_z}^2\right)$, $\mathbf{e} = (x, y, z)/r$ – the unit line-of-sight vector from the radar to the object based on the current coordinate estimate, and $V_{\mathrm{r,filtr}}$ is the filtered radial velocity. The method consists of three stages.

1) *Projection onto the radial-velocity constraint.* The vector **V** is corrected so as to satisfy the condition $\mathbf{e} \cdot \mathbf{V}' = V_{\mathrm{r,filtr}}$, while remaining as close as possible to the original estimate according to a weighted sum-of-squared-deviations criterion, with weights inversely proportional to the variances of the coordinate filter [21].

Formally, **V′** is defined as the solution of the constrained quadratic minimization problem

$$J(\mathbf{V}') = (\mathbf{V}' - \mathbf{V})^{\mathrm{T}} \mathbf{W}^{-1} (\mathbf{V}' - \mathbf{V}) \to \min \quad \text{subject to } \ \mathbf{e} \cdot \mathbf{V}' = V_{\mathrm{r,filtr}},$$

where $\mathbf{W}^{-1} = \mathrm{diag}\left(\sigma_{V_x}^{-2}, \sigma_{V_y}^{-2}, \sigma_{V_z}^{-2}\right)$ and the quadratic form $J(\mathbf{V}')$ penalizes the weighted squared deviation of each component of **V′** from **V**.

The problem is solved by the method of Lagrange multipliers – the same general approach as in other recent constrained-filtering formulations for vector-valued state estimates [22].

Introducing a Lagrange multiplier λ for the linear constraint, the unconstrained Lagrange function is

$$L(\mathbf{V}', \lambda) = (\mathbf{V}' - \mathbf{V})^{\mathrm{T}} \mathbf{W}^{-1} (\mathbf{V}' - \mathbf{V}) - 2\lambda(\mathbf{e} \cdot \mathbf{V}' - V_{\mathrm{r,filtr}}),$$

factor 2 and the sign of λ being chosen only for algebraic convenience and not affecting the result. A minimum of $J(\mathbf{V}')$ subject to the constraint is necessarily a stationary point of L, so setting the gradient with respect to **V′** to zero,

$$\nabla_{\mathbf{V}'} L = 2\mathbf{W}^{-1}(\mathbf{V}' - \mathbf{V}) - 2\lambda \mathbf{e} = 0,$$

gives $\mathbf{V}' = \mathbf{V} + \lambda \mathbf{W}\mathbf{e}$. Substituting this expression into the constraint $\mathbf{e} \cdot \mathbf{V}' = V_{\mathrm{r,filtr}}$ yields $\mathbf{e} \cdot \mathbf{V} + \lambda \mathbf{e}^{\mathrm{T}} \mathbf{W} \mathbf{e} = V_{\mathrm{r,filtr}}$, whence

$$\lambda = \frac{V_{\mathrm{r,filtr}} - \mathbf{e} \cdot \mathbf{V}}{\mathbf{e}^{\mathrm{T}} \mathbf{W} \mathbf{e}}.$$

Because $J(\mathbf{V}')$ is a positive-definite quadratic form (all variances $\sigma_{V_x}^2, \sigma_{V_y}^2, \sigma_{V_z}^2$ are finite and positive) and the constraint is linear, the stationary point found above is the unique global minimum of the problem; combining two preceding expressions for **V′** and λ gives the closed-form solution:

$$\mathbf{V}' = \mathbf{V} + \lambda \mathbf{W}\mathbf{e}, \quad \lambda = \frac{V_{\mathrm{r,filtr}} - \mathbf{e} \cdot \mathbf{V}}{\mathbf{e}^{\mathrm{T}} \mathbf{W}^{-1} \mathbf{e}}. \tag{2}$$

The result of this stage, **V′**, exactly satisfies the radial-velocity constraint regardless of how far the course of the original estimate **V** is from the true course – velocity components with lower variance (typically more robust to coordinate noise) are corrected less, and those with higher variance more.

2) *Reconciling the course from the track.* Independently of the coordinate filter, over a sliding window of the last N raw measurements (on the order of ten), the geometric course of the object is computed from the displacement between the first and last point of the window. Such an estimate is not subject to the fluctuations of the coordinate filter but is statistically reliable only when the apparent displacement speed within the window is sufficient; when it is not, the fallback course from stage 1 is used. The resulting course is smoothed exponentially and pulled toward the working course of the previous cycle. The horizontal speed magnitude at the new course is recomputed so as to exactly satisfy $V_{\mathrm{r,filtr}}$ constraint at the fixed vertical component from stage 1 – similarly to (2), but with respect to a single unknown. The solution is stable only given sufficient geometric sensitivity of the radial velocity to changes in horizontal speed magnitude at the given course (see below); when sensitivity is insufficient, or the solution falls outside a plausible range, the fallback value from stage 1 is used.

3) *Inertial smoothing.* The horizontal speed magnitude, course, and vertical component are smoothed exponentially with a limit on the maximum change per cycle (to suppress residual noise of individual

measurements), after which the result is re-projected onto the radial-velocity constraint using formula (2), since smoothing may have violated the exact equality $\mathbf{e} \cdot \mathbf{V}' = V_{r,\mathrm{filtr}}$.

The applicability condition of stage 2 requires separate consideration. At a fixed course ψ, the horizontal speed magnitude enters the expression for the radial velocity with a coefficient

$$k = e_x \sin\psi + e_z \cos\psi, \tag{3}$$

where e_x, e_z are horizontal components of the vector **e** along axes x and z, respectively, characterizing the sensitivity of V_r to changes in horizontal speed magnitude at the given course. As $|k| \to 0$ (a course close to perpendicular to the horizontal projection of the line of sight), the radial velocity becomes practically independent of the horizontal speed magnitude, and recovering the latter from V_r becomes unstable – when $|k|$ is insufficient, the solution of stage 2 is not constructed and the fallback value is used. This limitation is fundamental: it reflects the physical incompleteness of the radial velocity as a source of information about the full velocity vector, rather than a shortcoming of the particular implementation of the method.

Results of Simulation Modeling. The method's performance was verified on synthetic data for a hypothetical multifunction radar. Three typical scenarios of the true object trajectory, observed by the same radar (identical errors in range, azimuth, elevation angle, and radial velocity measurement), are considered: a low-speed helicopter, an airplane, and a ballistic object. For all three scenarios it was verified that the course does not approach the perpendicular to the radar-object line of sight throughout the trajectory (see |k| condition above). The root-mean-square measurement errors adopted are 20 m for range, $2 \cdot 10^{-3}$ rad for azimuth and elevation angle, and 1,0 m/s for radial velocity (an order of magnitude more accurate than differentiating noisy coordinates provides at the adopted update period of 0.1 s). The base coordinate filter is a three-dimensional Kalman filter; for the helicopter and airplane, whose maneuver is a turn, a nearly constant-velocity model is used (CV, which does not account for the maneuver [23; 24]); for the ballistic object, whose vertical acceleration component (gravity) is strictly constant while the horizontal component (aerodynamic deceleration) is not, but which is on the whole better approximated by a constant-acceleration model than by a zero-acceleration one, a higher-order filter is used – a nearly constant-acceleration model (CA [24]); independently, a filter of the same structure is used for the radial velocity from the scalar V_r measurement. For each scenario, 2000 realizations of statistical Monte Carlo simulation were carried out; the accuracy metric computed at each time step is the root-mean-square error (RMSE) over realizations for position, course, horizontal speed magnitude $|\mathbf{V}_{hor}|$, and full speed magnitude $|\mathbf{V}|$, as functions of time. Summary results are given in the table below; position is not included, since correction does not change it in any of the scenarios (see the Method section above).

RMSE of course and speed magnitude from Monte Carlo simulation results (2000 runs per scenario)

Scenario	RMSE of course ψ, deg		RMSE $|\mathbf{V}_{hor}|$, m/s		RMSE $|\mathbf{V}|$ (full magnitude), m/s	
	Before	After	Before	After	Before	After
Helicopter	17.06	8.10	7.79	2.68	7.83	2.74
Airplane	9.26	4.40	30.42	17.08	29.95	16.67
Ballistic object	1.46	1.58	22.10	14.12	21.18	14.43

Helicopter (low-speed). The true trajectory has a horizontal speed of 60 m/s with a deceleration of 1.5 m/s^2 and a comparatively sharp turn of 4 deg/s (range 4183–4853 m). The object displacement over a single update period (about 6 m) is comparable to the single-measurement coordinate error, satisfying condition (1); in addition, the low object speed means that the relative contribution of measurement noise to the coordinate filter's course and speed estimate is especially large. The Monte Carlo results are given in the table above; the RMSE time history (over 2000 realizations) is shown in Figure 2. In all three panels the gray curve is the base CV filter without correction, the red curve is the same filter with V_r-based correction, and the shaded band marks the instantaneous gap between them; both the sharp initial transient (the filter settling from its startup state) and the subsequent gap between the curves are visible in course and in both speed-magnitude panels alike, reflecting the turn that the CV model cannot track exactly. The same color and shading convention is used in Figures 3 and 4.

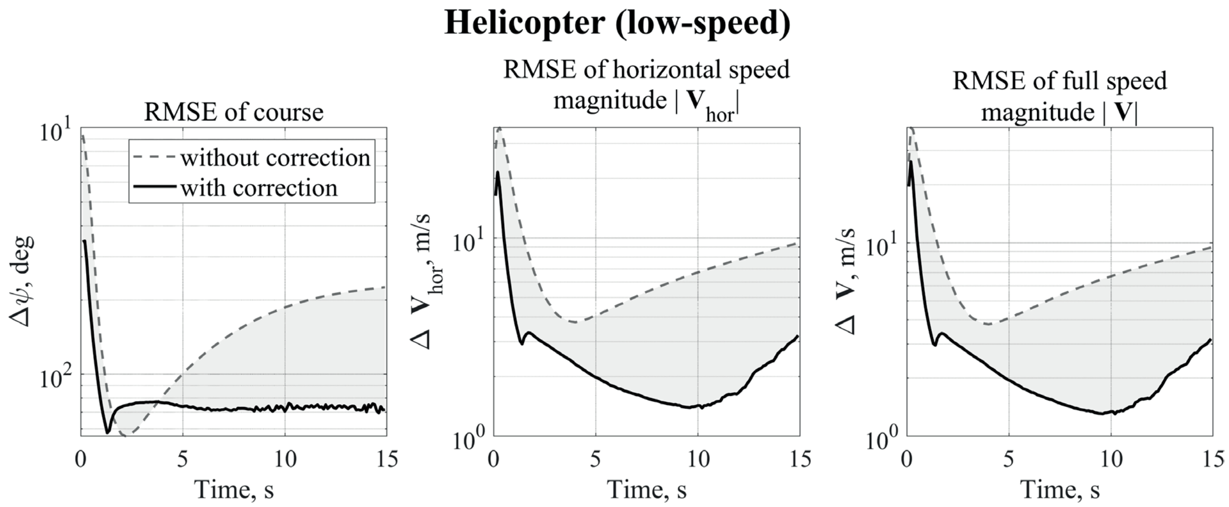


Fig. 2. Helicopter – RMSE of course, $|\mathbf{V}_{\text{hor}}|$, and $|\mathbf{V}|$ over the tracking interval

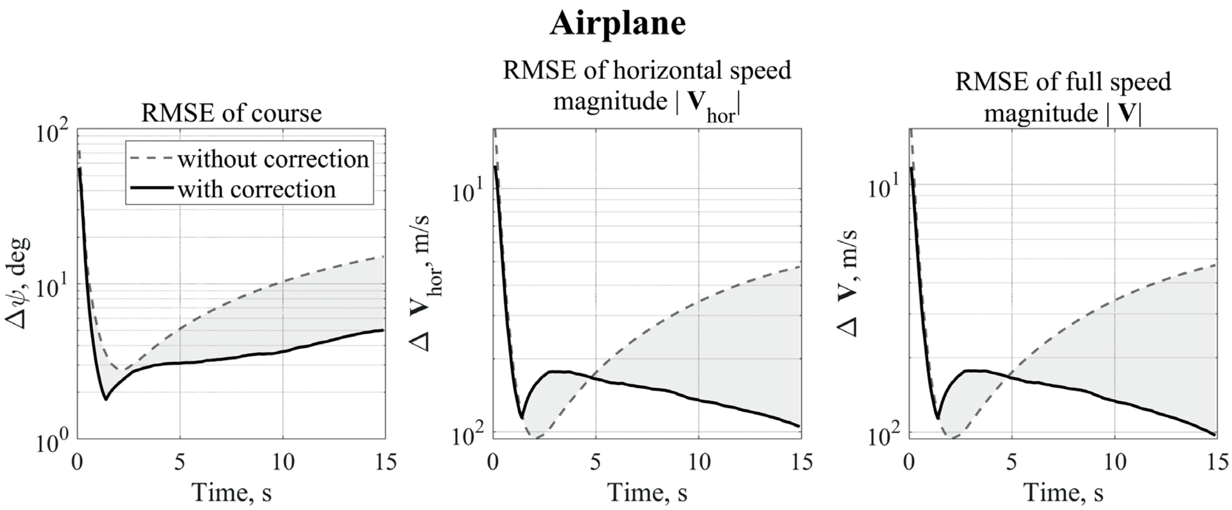


Fig. 3. Airplane – RMSE of course, $|\mathbf{V}_{\text{hor}}|$, and $|\mathbf{V}|$ over the tracking interval

Airplane. The true trajectory is a gentle turn with an angular rate of 2 deg/s and a linear deceleration of the horizontal speed magnitude from 220 to ~118 m/s over 15 s of observation (range 10440–12121 m). The base coordinate filter uses a nearly constant-velocity model that does not account for the maneuver, so its course and speed-magnitude estimates systematically lag behind the object's true turn and deceleration – the lag does not decrease as data accumulate but grows together with the maneuver (table above, Figure 3). The gap between the gray and red curves widens steadily over the 15 s interval in all three panels, rather than narrowing as more measurements accumulate. This is a sign of a systematic, rather than statistical, error, which can only be eliminated by correction, not longer-term observation.

Ballistic object. The true trajectory has an initial horizontal speed of 500 m/s, decreasing to ~287 m/s over 15 s under aerodynamic drag proportional to the square of speed ($dV/dt = -kV^2$, a more realistic deceleration model than constant acceleration – the drag force genuinely decreases in magnitude as speed drops: the horizontal acceleration here varies from ≈−25 to ≈−8 m/s² over the observation interval), with no turn in course, but with a constant vertical acceleration of −9.81 m/s^2 (gravity) (range 8500–11700 m). Unlike the helicopter and airplane, whose maneuver is a turn, here neither a nearly constant-velocity model (CV) nor a nearly constant-acceleration model (CA) describes the motion exactly: gravity along the vertical is a strictly constant acceleration, exactly matching the CA model, whereas the horizontal deceleration is not, since the true acceleration there continu-

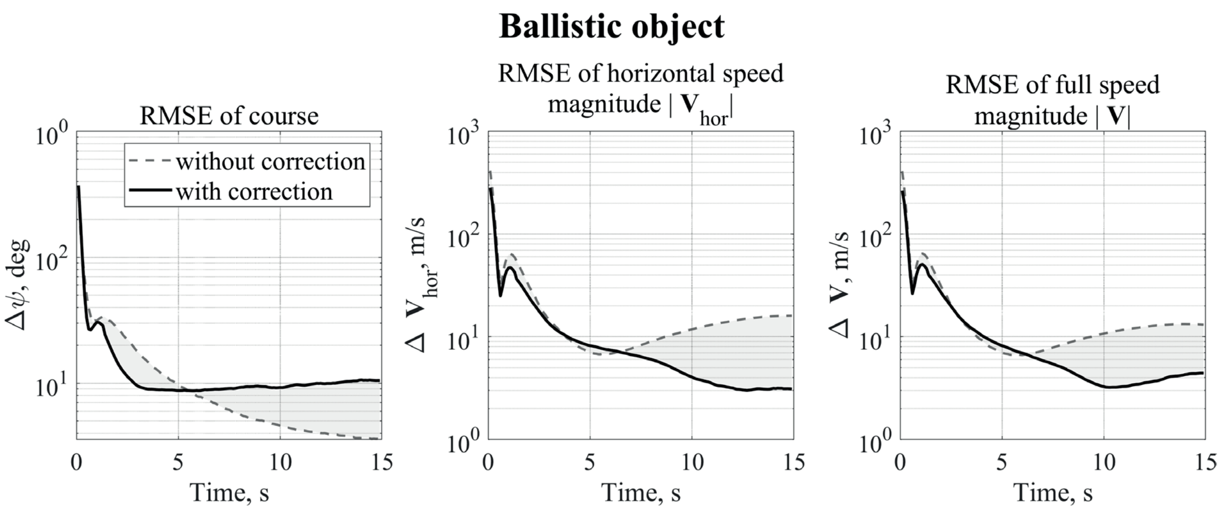


Fig. 4. Ballistic object – RMSE of course, $|\mathbf{V}_{\text{hor}}|$, and $|\mathbf{V}|$ over the tracking interval

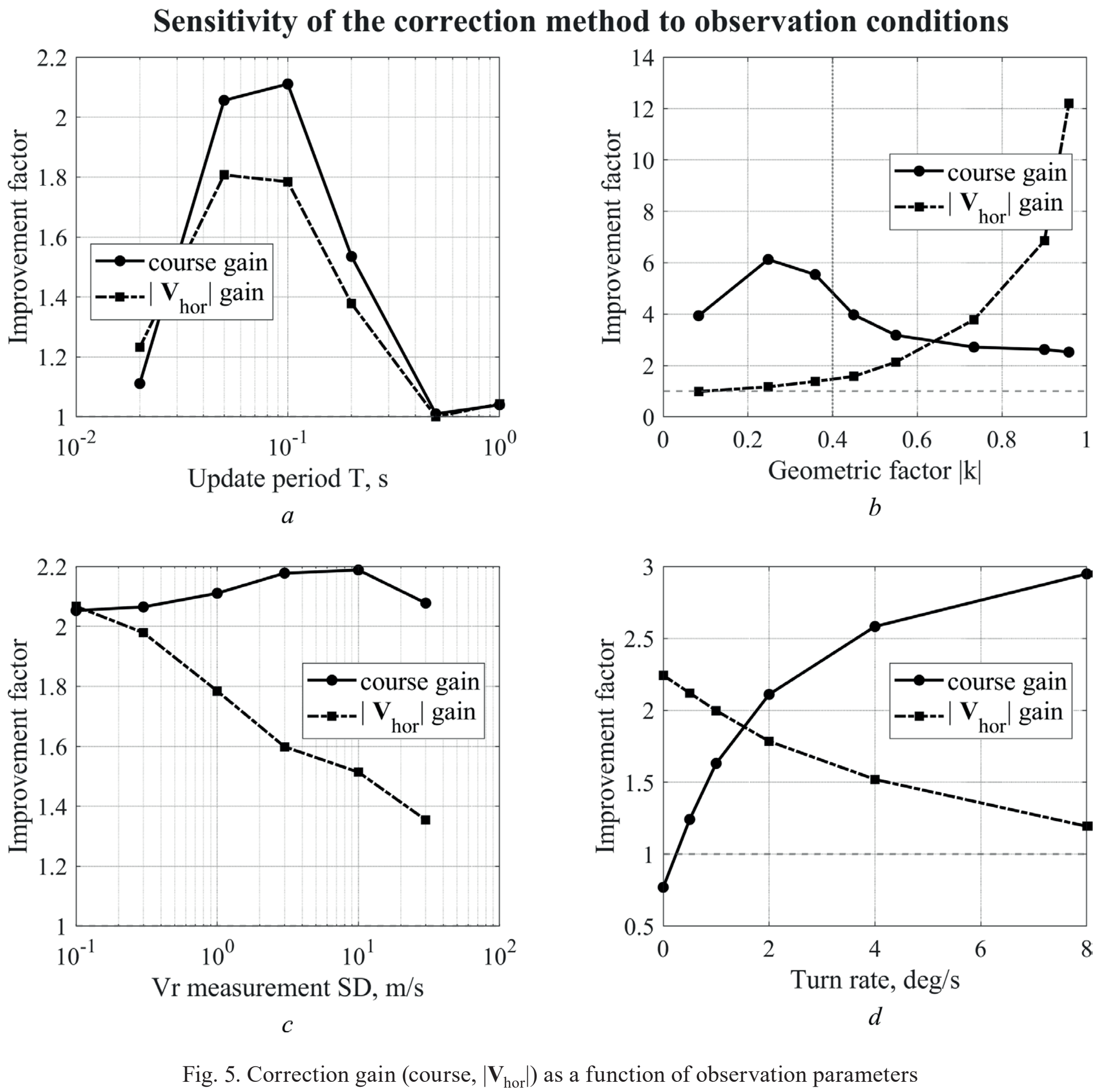
ously decreases in magnitude. Nevertheless, CA approximates the motion substantially better than CV – therefore, for this scenario, the base coordinate filter is taken of higher order (CA) rather than constant-velocity (CV), as for the other two scenarios.

The base CA filter tracks the vertical component exactly (gravity is strictly constant), but systematically lags behind the horizontal deceleration, which decreases in magnitude – a mismatch of the same type as that of the CV model with a turn for the helicopter and airplane: the model extrapolates acceleration as constant, whereas it is in fact decreasing. Correction by V_{r} uses the fact that the radial velocity is measured independently and accurately reflects the true deceleration, which is not extrapolated from a fixed acceleration: $|\mathbf{V}_{\text{hor}}|$ RMSE decreases from 22.10 to 14.12 m/s (by a factor of 1.6), and the full-magnitude $|\mathbf{V}|$ RMSE – from 21.18 to 14.43 m/s (by a factor of 1.5) (table above, Figure 4). Moreover, as for the helicopter and airplane, the benefit grows over time as the systematic error of the uncorrected CA filter accumulates. The course here, unlike the speed magnitude, remains accurate even without correction (the true course does not change), so the course-correction stage (stage 2, Method section) gives no benefit and only slightly increases the RMSE owing to the noise of the geometric track estimate: 1.46 → 1.58 deg. This is consistent with the general principle of the method: correction compensates specifically for the systematic mismatch between the base filter's model and the object's true motion – here it is present in speed magnitude (and correction gives a genuine benefit) and is practically absent in course (and correction gives no benefit). This asymmetry between panels is the key feature to note in the figure: the middle and bottom panels show the familiar widening gap between the gray and red curves, while the top panel (course) shows two curves tracking each other closely throughout, with the red curve never below the gray one.

The position error is not changed by correction in any of three scenarios – the method, as noted above, does not affect the coordinate estimate or its covariance, working only with the velocity vector. Three scenarios together present a complementary picture: for the helicopter and airplane, whose maneuver (a turn) is not described by the base filter's nearly constant-velocity (CV) model, correction improves both course and speed magnitude; for the ballistic object, whose vertical acceleration component is described exactly by the higher-order filter (CA), while the horizontal component is described only approximately (the true deceleration is not constant), correction gives a genuine benefit in speed magnitude (where the mismatch is present) but not in course (where it is absent – the course is accurate even without correction). In all four cases where a mismatch between the filter model and the motion is present (course and speed for the helicopter/airplane, speed for the ballistic object), the benefit grows over time together with the accumulating RMSE of the uncorrected filter. This directly confirms the method's general principle: it corrects specifically the velocity-vector components along which the object trajectory maneuvers beyond the model built into the base coordinate filter [23; 24] – the effectiveness of the correction is determined by the degree of mismatch between this model and the object's true motion, not by the presence of a maneuver as such.

Sensitivity of the Method to Observation Parameters. Besides object type, the correction's effectiveness depends on the observation conditions – the update period, the observation geometry, the accuracy of the radial velocity measurement, and the maneuver intensity. To quantify this, each parameter was varied individually while holding the other conditions fixed (an Airplane-like scenario, 2000 Monte Carlo runs per point); the results are shown in Figure 5.

The update period *T* was varied from 0.02 to 1.0 s at a constant observation duration (15 s). The gain in course and in $|\mathbf{V}_{hor}|$ peaks in the range T = 0.05–0.1 s (a factor of 1.8–2.1) and decreases in both directions: as *T* increases beyond ≈0.2 s, the object displacement per cycle ceases to be small relative to the coordinate noise (condition (1)), and the coordinate filter alone estimates the velocity accurately enough – the gain tends to 1 (T = 0.5–1.0 s, gain is 1.00–1.04). As *T* decreases below ≈0.05 s the gain also drops (to 1.11–1.23 at T = 0.02 s) – here the fixed length of the track-based course-estimation window (N = 15 measurements, see the Method section) comes into play: at a high data-output rate, the same number of measurements spans a substantially shorter time interval, the object displacement within the window shrinks proportionally to *T*, and the geometric course estimate becomes statistically less reliable. This refines the method's applicability condition: besides *T* being small relative to $\sigma/|\mathbf{V}|$, the track-



a *b* *c* *d*

Fig. 5. Correction gain (course, $|\mathbf{V}_{hor}|$) as a function of observation parameters (2000 Monte Carlo runs per point)

based course-estimation window must also span a sufficient time interval, not merely a fixed number of measurements.

The geometric factor $|k|$ was varied by changing the object's course relative to the radar-object line of sight, at the Airplane baseline's turn rate of 2 deg/s (held fixed, as for the other panels), from 0.96 (course nearly along the line of sight) to 0.08 (course nearly perpendicular to it). The gain in $|\mathbf{V}_{\mathrm{hor}}|$ decreases monotonically from the factor of 12.20 at $|k| = 0.96$ to approximately 1 (0.99) at $|k| = 0.08$ – a direct quantitative confirmation of condition (3): the radial velocity loses sensitivity to the horizontal speed magnitude as the course approaches the perpendicular to the line of sight, and the benefit vanishes almost exactly at that geometric limit. The gain in course over the same range is now a genuine benefit throughout – from the factor of 2.52 at $|k| = 0.96$, up to 6.13 near $|k| = 0.25$, and 3.94 at the most oblique offset ($|k| = 0.08$): since the constant turn is present at every point of this sweep (unlike the case in Figure 5 at $\omega = 0$), the course-correction stage has a genuine mismatch to correct throughout $|k|$ range as well.

The accuracy of the radial-velocity measurement σ_{V_r} was varied from 0.1 to 30 m/s (baseline value is 1.0 m/s). The gain in $|\mathbf{V}_{\mathrm{hor}}|$ decreases monotonically from the factor of 2.07 at $\sigma_{V_\mathrm{r}} = 0.1$ m/s to 1.35 at $\sigma_{V_\mathrm{r}} = 30$ m/s (30 times worse than baseline) – but even at this substantially degraded V_r accuracy, the gain does not vanish entirely over the tested range, since the independence of V_r measurement from the coordinate noise is itself useful information. The gain in course, by contrast, is nearly independent of σ_{V_r} (2.05–2.19 over the entire range) – the course estimate (stage 2, Method section) is built from the track geometry and does not use V_r directly.

Maneuver intensity (turn rate ω, Figure 5) was varied from 0 (no turn) to 8 deg/s at the constant horizontal deceleration of 7 m/s^2 (the Airplane baseline, held fixed to isolate the effect of ω alone). The gain in course grows monotonically with ω – from 0.77 (a slight degradation at $\omega = 0$, where there is no turn and the course-correction stage has nothing to fix) to the factor of 2.95 at $\omega = 8$ deg/s. The gain in $|\mathbf{V}_{\mathrm{hor}}|$, by contrast, decreases monotonically over the same range – from 2.24 at $\omega = 0$ (a pure deceleration mismatch, the same regime as the ballistic object) to 1.19 at $\omega = 8$ deg/s: as the turn comes to dominate the model mismatch, a growing share of the correction's effect is spent on course rather than speed magnitude, so the relative benefit to $|\mathbf{V}_{\mathrm{hor}}|$ shrinks. Importantly, the gain in $|\mathbf{V}_{\mathrm{hor}}|$ never drops below 1 over the entire tested range: since a genuine deceleration mismatch is present throughout (unlike in a zero-deceleration case), the correction is beneficial here as well, consistent with the object scenarios.

All four sweeps are collected in Figure 5: in every panel, the dashed horizontal line marks the gain of 1 (no effect of correction), so a curve above it is a genuine benefit and a curve below it is a (mild) degradation; the dotted vertical line in panel (b) marks the geometric-sensitivity threshold $|k| = 0.4$ below which stage 2 of the method (see the Method section above) no longer constructs a solution.

Taken together, the results in Figure 5 quantitatively confirm the method's applicability conditions formulated in the Problem Statement and Method sections (smallness of the object displacement per cycle relative to the coordinate noise, sufficient geometric sensitivity $|k|$, V_r accuracy sufficient to exceed the coordinate filter's noise), and refine them with the additional observation that the track-based course-estimation window must span a sufficient time interval, not merely a fixed number of measurements.

Conclusion. A method has been proposed for correcting the velocity vector of the tracking filter of a multifunction radar using an independently filtered radial-velocity estimate, applicable in a continuous-tracking mode with a high data-output rate, when the object displacement over an update period is comparable to the single-measurement error in coordinates. The method is based on an exact weighted projection of the velocity vector onto the constraint imposed by the radial velocity, reconciliation of the course with an independent geometric estimate from the track, and inertial smoothing of the result; the coordinates and their covariance in the coordinate filter are not changed.

From the results of Monte Carlo simulation on synthetic data for a hypothetical radar, for three typical scenarios (helicopter, airplane, ballistic object), the method reduces the RMSE of course and horizontal speed magnitude for the helicopter and airplane, whose maneuver (a turn) is not described by the base filter's constant-velocity model; for the ballistic object, whose base filter is of higher order (CA), correction gives a genuine benefit where the model mismatch persists (speed magnitude – owing to non-constant aerodynamic drag), and no benefit where the mismatch is absent (course, which does not change for the ballistic object). This confirms that the method's effectiveness is determined by the degree of mismatch between the filter model and the object's true motion, not by the presence of a maneuver as such.

The applicability condition of the method for recovering the horizontal speed magnitude has also been shown and explicitly formulated, related to the geometric sensitivity of the radial velocity to changes in speed magnitude at a given object course relative to the line of sight. The sensitivity of the gain to the update period, geometric factor, radial-velocity measurement accuracy, and maneuver intensity was investigated quantitatively; in particular, it was shown that the track-based course-estimation window must be sufficient in duration, not merely in the number of measurements.

Further research is intended to be directed toward experimental verification of the method on data from real hardware, quantitative assessment of the influence of the smoothing parameters on the stability and speed of convergence of the correction, and the possibility of adaptively tuning these parameters [17] by analogy with the adaptive schemes used for managing the resources of multifunction radars [2–4].

From a practical standpoint, the method is attractive because it does not require redesigning the base coordinate filter: correction is applied as a lightweight closed-form post-processing step, computed once per update cycle without iterative optimization, and is compatible with coordinate filters of different orders (CV, CA), as demonstrated by three scenarios considered here. This makes the approach suitable for real-time implementation within existing multifunction radar tracking software, requiring only that an independently filtered radial-velocity estimate already be available from the Doppler channel, as in the case of the continuous-tracking mode considered throughout this paper.

Acknowledgements. This work was carried out with the support of a grant from the Belarusian Republican Foundation for Fundamental Research (project no. T26УЗБ-007) and the Ministry of Higher Education, Science and Innovation of the Republic of Uzbekistan (project no. FL-10425067114).

Благодарности. Работа выполнена при поддержке гранта Белорусского республиканского фонда фундаментальных исследований (проект № Т26УЗБ-007) и Министерства высшего образования, науки и инноваций Республики Узбекистан (проект № FL-10425067114).

References

1. Blackman S. S., Popoli R. *Design and Analysis of Modern Tracking Systems*. Norwood, MA, Artech House Publ., 1999. 1230 p.

2. Shi C., Wang Y., Salous S., Zhou J., Yan J. Joint Transmit Resource Management and Waveform Selection Strategy for Target Tracking in Distributed Phased Array Radar Network. *IEEE Transactions on Aerospace and Electronic Systems*, 2022, vol. 58, no. 4, pp. 2762–2778. https://doi.org/10.1109/TAES.2021.3138869

3. Hashmi U. S., Akbar S., Adve R., Moo P. W., Ding J. Artificial Intelligence Meets Radar Resource Management: A Comprehensive Background and Literature Review. *IET Radar, Sonar & Navigation*, 2023, vol. 17, no. 2, pp. 153–178. https://doi.org/10.1049/rsn2.12337

4. Shi C., Tang Z., Ding L., Yan J. Multidomain Resource Allocation for Asynchronous Target Tracking in Heterogeneous Multiple Radar Networks with Nonideal Detection. *IEEE Transactions on Aerospace and Electronic Systems*, 2024, vol. 60, no. 2, pp. 2016–2033. https://doi.org/10.1109/TAES.2023.3347214

5. Blom H. A. P., Bar-Shalom Y. The Interacting Multiple Model Algorithm for Systems with Markovian Switching Coefficients. *IEEE Transactions on Automatic Control*, 1988, vol. 33, no. 8, pp. 780–783. https://doi.org/10.1109/9.1299

6. Lee I. H., Park C. G. An Improved Interacting Multiple Model Algorithm with Adaptive Transition Probability Matrix Based on the Situation. *International Journal of Control, Automation and Systems*, 2023, vol. 21, no. 10, pp. 3299–3312. https://doi.org/10.1007/s12555-022-0989-4

7. Choi Y. K., Lee I. H., Park C. G. Robust Adaptive Transition Probability Matrix in Interacting Multiple Model with Polynomial Functions and Feedback Structure. *International Journal of Control, Automation and Systems*, 2024, vol. 22, no. 12, pp. 3547–3558. https://doi.org/10.1007/s12555-024-0500-5

8. Wu X., Chen Y., Li Z., Hong Z., Hu L. EFEAR-4D: Ego-Velocity Filtering for Efficient and Accurate 4D Radar Odometry. *IEEE Robotics and Automation Letters*, 2024, vol. 9, no. 11, pp. 9828–9835. https://doi.org/10.1109/LRA.2024.3466071

9. Zhao X., Zhao X., Liu Z., Zhang W. A Method to Track Moving Targets Using a Doppler Radar Based on Converted State Kalman Filtering. *Electronics*, 2024, vol. 13, no. 8, art. ID 1415. https://doi.org/10.3390/electronics13081415

10. Pan Z., Ding F., Zhong H., Lu C. X. *RaTrack: Moving Object Detection and Tracking with 4D Radar Point Cloud. Arxiv* [Preprint], 2023. Available at: https://arxiv.org/abs/2309.09737. https://doi.org/10.48550/arXiv.2309.09737

11. Lopez Fernandez S., Samarasekera A. C. J., Feger R., Stelzer A. Velocity Vector Estimation in Automotive Radar Networks. *International Journal of Microwave and Wireless Technologies*, 2025, vol. 17, no. 2, pp. 347–356. https://doi.org/10.1017/S175907872400117X

12. Wang J., Liu A., Yu C. Long-Term Tracking for High-Frequency Surface Wave Radar Based on Heading Constraint Filtering Combined ELM. *Measurement Science and Technology*, 2024, vol. 35, no. 12, art. ID 126313. https://doi.org/10.1088/1361-6501/ad7be5

13. Solonar A. S., Khmarski P. A., Tsuprik S. V. Tracking Estimator of the Ground Target Coordinates and Motion Parameters Using Onboard Optical Location System Data. *Gyroscopy and Navigation*, 2023, vol. 14, no. 3, pp. 244–258. https://doi.org/10.1134/S2075108723030082

14. Solonar A. S., Khmarski P. A. Main Problems of Trajectory Processing and Approaches to Their Solution within the Framework of Multitarget Tracking. *Journal of Physics: Conference Series*, 2021, vol. 1864, no. 1, art. ID 012004. https://doi.org/10.1088/1742-6596/1864/1/012004

15. Khmarskiy P. A., Solonar A. S., Naumov A. O. Device of tracking processing for indicator channel of short-range radio navigation systems. *Vestsi Natsyyanal'nai akademii navuk Belarusi. Seryya fizika-technichnych navuk = Proceedings of the National Academy of Sciences of Belarus. Physical-technical series*, 2023, vol. 68, no. 1, pp. 60–71 (in Russian). https://doi.org/10.29 235/1561-8358-2023-68-1-60-71

16. Artemyev V. M., Khmarski P. A., Naumov A. O. Reducing Radar Measurement Errors Using Ionosphere and Magnetosphere Monitoring Data. *Doklady BGUIR*, 2025, vol. 23, no. 4, pp. 54–62 (in Russian). https://doi.org/10.35596/1729-7648-2025-23-4-54-62

17. Khmarskiy P. A. Generalized technique for optimizing the parameters of tracking estimators of coordinates and motion parameters in air and ground situation monitoring systems. *Vestsi Natsyyanal'nai akademii navuk Belarusi. Seryya fizika-technichnych navuk = Proceedings of the National Academy of Sciences of Belarus. Physical-technical series*, 2025, vol. 70, no. 2, pp. 159–165. https://doi.org/10.29235/1561-8358-2025-70-2-159-165

18. Tsuprik S. V., Solonar A. S., Khmarski P. A. Statistical synthesis of a bayesian algorithm for image segmentation and measurement of aerial object coordinates. *Vestsi Natsyyanal'nai akademii navuk Belarusi. Seryya fizika-technichnych navuk = Proceedings of the National Academy of Sciences of Belarus. Physical-technical series*, 2026, vol. 71, no. 1, pp. 57–66. https://doi.org/10.29235/1561-8358-2026-71-1-57-66

19. Khmarski P. A. *Processing of Measurement Results of Coordinates and Motion Parameters in Air and Ground Situation Monitoring Systems*. Minsk, Belaruskaya navuka Publ., 2025. 261 p. (in Russian).

20. Solonar A. S., Khmarski P. A., Naumov A. O., Juraev D. A., Muxammedov B. M. The Use of Numerical Monte Carlo Integration to Verify the Physical Feasibility of a Trajectory Based on Surveillance Radar Data. *Stochastic Modelling and Computational Sciences*, 2023, vol. 3, no. 1, pp. 59–73. https://doi.org/10.61485/SMCS.27523829/v3n1P5

21. Bar-Shalom Y., Li X. R., Kirubarajan T. *Estimation with Applications to Tracking and Navigation: Theory, Algorithms and Software*. New York, John Wiley & Sons Publ., 2001. 584 p. https://doi.org/10.1002/0471221279

22. Wen Z., Zheng L., Zeng T. Extended Object Tracking Using an Orientation Vector Based on Constrained Filtering. *Remote Sensing*, 2025, vol. 17, no. 8, art. ID 1419. https://doi.org/10.3390/rs17081419

23. Wang J., Wang X., Chen Y., Yan M., Lan H. Model Adaptive Kalman Filter for Maneuvering Target Tracking Based on Variational Inference. *Electronics*, 2025, vol. 14, no. 10, art. ID 1908. https://doi.org/10.3390/electronics14101908

24. Li X. R., Jilkov V. P. Survey of Maneuvering Target Tracking. Part I: Dynamic Models. *IEEE Transactions on Aerospace and Electronic Systems*, 2003, vol. 39, no. 4, pp. 1333–1364. https://doi.org/10.1109/TAES.2003.1261132

Information about the authors

Petr A. Khmarski – Ph. D. (Engineering), Associate Professor, Leading Researcher, Institute of Applied Physics of the National Academy of Sciences of Belarus (16, Akademicheskaya Str., 220072, Minsk, Republic of Belarus). E-mail: pierre2009@mail.ru

Valentin M. Artemiev – Corresponding Member of the National Academy of Sciences of Belarus, Dr. Sc. (Engineering), Professor, Chief Researcher, Institute of Applied Physics of the National Academy of Sciences of Belarus (16, Akademicheskaya Str., 220072, Minsk, Republic of Belarus). E-mail: artemiev@iaph.bas-net.by

Alexander O. Naumov – Ph. D. (Physics and Mathematics), Head of the Laboratory, Institute of Applied Physics of the National Academy of Sciences of Belarus (16, Akademicheskaya Str., 220072, Minsk, Republic of Belarus). E-mail: naumov@iaph.bas-net.by

Информация об авторах

Хмарский Петр Александрович – кандидат технических наук, доцент, ведущий научный сотрудник, Институт прикладной физики Национальной академии наук Беларуси (ул. Академическая, 16, 220072, Минск, Республика Беларусь). E-mail: pierre2009@mail.ru

Артемьев Валентин Михайлович – член-корреспондент Национальной академии наук Беларуси, доктор технических наук, профессор, главный научный сотрудник, Институт прикладной физики Национальной академии наук Беларуси (ул. Академическая, 16, 220072, Минск, Республика Беларусь). E-mail: artemiev@iaph.bas-net.by

Наумов Александр Олегович – кандидат физико-математических наук, заведующий лабораторией, Институт прикладной физики Национальной академии наук Беларуси (ул. Академическая, 16, 220072, Минск, Республика Беларусь). E-mail: naumov@iaph.bas-net.by

Sergey V. Tsuprik – Ph. D. (Engineering), Associate Professor, Military Academy of the Republic of Belarus (220, Nezavisimosti Ave., 220057, Minsk, Republic of Belarus). E-mail: Serhio.Observer@yandex.ru

Bobomurod M. Muxammedov – Head of the Research Laboratory, Institute of Military Aviation of the University of Military Security and Defense of the Republic of Uzbekistan (54, Jayhun Str., 180117, Qarshi, Republic of Uzbekistan). E-mail: bobomurod_1983@inbox.ru

Davron A. Juraev – Ph. D. (Physics and Mathematics), Associate Professor, Leading Researcher, Institute of Military Aviation of the University of Military Security and Defense of the Republic of Uzbekistan (54, Jayhun Str., 180117, Qarshi, Republic of Uzbekistan). E-mail: juraevdavron12@gmail.com

Цуприк Сергей Викторович – кандидат технических наук, доцент, Военная академия Республики Беларусь (пр. Независимости, 220, 220057, Минск, Республика Беларусь). E-mail: Serhio.Observer@yandex.ru

Мухаммедов Бобомурод Мухаммадкаримович – начальник научно-исследовательской лаборатории, Институт военной авиации Университета военной безопасности и обороны Республики Узбекистан (ул. Джайхун, 54, 180117, Карши, Республика Узбекистан). E-mail: bobomurod_1983@inbox.ru

Жураев Даврон Аслонкулович – кандидат физико-математических наук, доцент, ведущий научный сотрудник, Институт военной авиации Университета военной безопасности и обороны Республики Узбекистан (ул. Джайхун, 54, 180117, Карши, Республика Узбекистан). E-mail: juraevdavron12@gmail.com